A review of non-hydrostatic numerical models for the atmosphere

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Abstract. This review describes development of non-hydrostatic numerical models of the atmosphere for both operational and research purposes. Three major components of non-hydrostatic models are discussed. These include: governing equations in Section 1, numerical techniques to solve them in Section 2, and the third section describes inclusion of nonlinear physical processes. In Section 1, we describe the various approximations to the governing equations that are being used in nonhydrostatic models. The Boussinesq, anelastic, and fully compressible treatments are compared. Various vertical coordinate systems are also discussed. In Section 2, for the fully compressible non-hydrostatic equations, we review various techniques to handle acoustic and fast moving gravity modes, including semi-implicit and split-explicit time integration schemes to control the acoustic modes. Methods of defining vertical and lateral boundary conditions are also discussed. In Section 3, we review the inclusion of physical processes in various models and discuss data initialization procedures. In the last section, we summarize current status of the development of non-hydrostatic models with a brief comment on future research in this area.

1991 Mathematics Subject Classification: 86A10, 76Q05.

1. Introduction

Meso β and γ scale motions with characteristic length scales between 10 to 100 km in the atmosphere, such as mountain waves, thunderstorms, squall lines and convective motions, deviate from hydrostatic balance and therefore require the full three-dimensional equations of motion for their description. The full set of equations of motion are too complex to solve analytically, and approximate solutions are usually obtained numerically. Fully compressible Navier-Stokes equations not only allow gravity modes and meteorologically significant rotational modes but also allow acoustic modes which propagate at phase speeds about 300 m/s in both horizontal and vertical directions. These acoustic modes contain very little energy and meteorologically unimportant. However, their presence place a very severe limitation on the time steps that can be used for numerical integration of the model. For example, the maximum time steps one can use in the model must satisfy the Courant-Friedrichs-Lewy (CFL) condition, $\Delta t \leq \Delta s/C_s$, Δs is the spatial increment, C_s is the phase speed of the fastest moving acoustic wave, Δt is the maximum time increment one can use. If the vertical spatial increment is 30 m, a common value used in the planetary boundary modeling, a 0.1 second time step is needed. Early efforts in the development of non-hydrostatic models are concentrated in eliminating these acoustic modes by making some approximations to the governing equations. Most noteworthy of these efforts is by Ogura and Phillips [35] who proposed the so-called anelastic (sound-proof) approximations, which eliminate the acoustic waves completely from the model, yet the model is still able to describe the non-hydrostatic process. There are some modifications of the original anelastic approximations (e.g. Dutton and Fichtl [11]; Gough [16]; Wilhelmson and Ogura [45]; Lipps and Hemler [28]; Durran [9]; Lipps [27]). The anelastic approx*imations are widely used in many non-hydrostatic models for research purposes* (e.g. Clark [3]; Lipps and Hemler [29]; Kapitza [21]; Hemler et al. [18]; Kogan [26]). Since the middle 70's, most of the non-hydrostatic models developed are based on the fully compressible Navier-Stokes equations (e.g. Hill [19]; Tapp and White [43]; Klemp and Wilhelmson [24]; Cotton and Tripoli [4]). With the availability of high speed computers and efficient new numerical schemes, such as semi-implicit and split-explicit time integration techniques, one can run a fully compressible non-hydrostatic model as efficiently as it would be for a hydrostatic models. Several recent studies have demonstrated the feasibility of performing large scale flow simulations with non-hydrostatic models (e.g. Golding [15]; Cullen [5]; Tanguay et al. 42]).

In the following chapters three major components of non-hydrostatic model are discussed. Fully compressible Navier-Stokes equations in various vertical coordinate systems that are currently being used in various non-hydrostatic models are described in Section 2.1. Section 3.1 briefly describes various approximations that are made in the governing equations to eliminate acoustic modes. Section 3.2 describes a few time integration techniques that can be used to overcome the time step limitations imposed by the presence of acoustic modes in fully compressible set of equations. Inclusion of physical processes and data initialization techniques as well as the four-dimensional data assimilation that are suitable for non-hydrostatic models are also briefly discussed. There is an excellent paper written by Skamarock [41] on this subject.

2. Governing equations

2.1. Fully compressible non-hydrostatic equations

The governing equations for the atmospheric motions are the fully compressible Navier-Stokes equations, which are sufficiently general to be applicable to general circulation and synoptic scale motions as well as a wide range of mesoscale phenomena. We can write the fully compressible Navier-Stokes equations in a coordinate system with Z as the vertical coordinate as,

$$\frac{D\mathbf{V}}{Dt} = -2\Omega \times \mathbf{V} - \frac{1}{\rho} \nabla p + \mathbf{g} + \mathbf{F} \tag{2.1}$$

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{V} \tag{2.2}$$

$$\frac{DT}{Dt} = -\frac{RT}{C_v} \nabla \cdot \mathbf{V} + \frac{\dot{Q}}{C_v} \tag{2.3}$$

$$p = \rho RT. \tag{2.4}$$

Equation (2.1) to (2.4) are momentum, continuity, thermodynamic and equation of state, respectively. The total derivative, D/Dt, is defined as,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla.$$

The dependent variable $\mathbf{V}, \rho, T, p$ in equations (2.1) to (2.4) represent velocity, density, temperature and pressure, respectively. $\mathbf{F}$ is the frictional force, $\dot{Q}$ is the diabatic heat source. Most of the non-hydrostatic models use variations of these equations above for the computational convenience and meteorological phenomena of interest. It is very common to replace temperature with potential temperature, θ, and pressure with Exner function, Π, which are defined as,

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{C_p}}$$

$$\Pi = \left(\frac{p_0}{p}\right)^{\frac{R}{C_p}}.$$

The main advantage of replacing the temperature and pressure with the potential temperature and the Exner function is that the density, a non-observational variable, is not explicitly appear in the governing equations. This replacement is only for the computational convenience (e.g., Tapp and White [43]; Cotton and Tripoli [4]; Klemp and Wilhelmson [24]). Recently, Tanguay et al. [42] used another nondimensional pressure variable $q = ln(p/p_0)$ in their model. Using the new variable q, they were able to eliminate the density, such that the pressure gradient terms becomes $RT \nabla q$. With these modifications, the governing equation (2.1) to (2.4) can be written as (Tapp and White [43]),

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + f\mathbf{k} \times \mathbf{V} + g\mathbf{k} + \frac{1}{\rho} \nabla p = \mathbf{F} \tag{2.5}$$

$$\frac{\partial \rho}{\partial t} + \mathbf{V} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{V} = 0 \tag{2.6}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{V} \cdot \nabla \theta = Q\theta/(c_p T) \tag{2.7}$$

where $\mathbf{V} = (u, v, w)$ is the velocity vector in three dimensions and kk is the unit vector in the z-direction. The terms $\mathbf{F} = (Fx, Fy, Fz)$ describe the sources and sinks of the momentum in x, y, and z directions, respectively. The term Q represents the source and sink of heat. As suggested by Tapp and White [43], the governing equations above exclude some effects of the earth's rotation, the most important of which is probably the vertical component of the Coriolis force which may be large enough to contribute to non-hydrostatic effects in regions of strong zonal flow.

2.2. The vertical coordinate systems

Two types of vertical coordinate systems have been used in the non-hydrostatic models in the past. The first group is the geometric height and geometric height-based terrain following coordinates. The second group is the pressure and pressure-based terrain following coordinate. We discuss them in the following section.

The natural choice for the vertical coordinate would be the geometric height coordinate or the z-coordinate, if the ground surface is flat (e.g. 43]; Tanguay et al. [42]). When the topography is present in the model, a geometric height based terrain-following coordinate transformation is commonly used to deal with the difficulties at lower boundary that arise from using the conventional z-coordinate. Galchen and Somerville [14] proposed the following coordinate transformation

$$\bar{z} = H(z - z_s)/(H - z_s)$$

where $z_s(x, y)$ is the topographic height and H is the domain height. This coordinate transformation has been adopted in many non-hydrostatic as well as hydrostatic mesoscale models (e.g. Clark [3]). The height above the ground surface was also used as the vertical coordinate (Carpenter [2]).

Since most of the observations are made on pressure surfaces, however, it is convenient to use pressure as vertical coordinate. Using a rigorous scale analysis, Miller [32] and Miller and Pearce [33] have developed non-hydrostatic models with pressure as the vertical coordinate. Recently, several non-hydrostatic models are developed with a nondimensional vertical coordinate based on surface pressure (e.g. Xue and Thorpe [46]). The reader is referred to recent papers by Miller and White [31], and White [44] for more information about how to use pressure based terrain following coordinates in a non-hydrostatic model.

3. Numerical methods

The fully compressible non-hydrostatic governing equations discussed in the previous section allow the existence of full spectrum of wave motions, including the high frequency acoustic waves. Since acoustic waves contain very small amounts of energy, they are not meteorologically important. Because of their high frequency, however, acoustic waves restrict the time increment to a fraction of one can use if they are eliminated. Usually there are two methods to deal with the acoustic modes. One can increase computational efficiency of non-hydrostatic models either eliminating the acoustic modes from the model atmosphere by modifying the governing equations or computing the terms governing acoustic waves either implicitly or by using time-split techniques.

3.1. Physical approximations

There are two physical approximations that are commonly used in numerical models to eliminate the acoustic waves from the model atmosphere. One is hydrostatic assumption where the vertical accelerations are very small compared to other terms in the momentum equation for vertical component of the velocity. With this approximation, the equation of motion for vertical component of velocity reduces to hydrostatic relation. Another is anelastic approximations proposed by Ogura and Phillips [35].

3.1.1. Hydrostatic assumption. The hydrostatic assumption states that the acceleration of vertical velocity is small compared to other terms in the equation of the vertical component of the velocity therefore, this equation reduces to

$$\frac{\partial p}{\partial z} = -\rho g.$$

According to scale analysis, the hydrostatic approximation is valid when the aspect ratio (vertical scale of the motions/horizontal scale of the motions) is much less than unity. Vertically propagating acoustic waves are eliminated under hydrostatic assumption. For meso-α, synoptic and global scale circulations hydrostatic approximation is valid. The finest model resolution used at major weather forecasting centers, is about 40 km in horizontal. The use of hydrostatic approximation can be justified at these model resolutions. However, with the rapid increase in computer power, the model resolution has been steadily increasing. Eventually, it will be necessary to relax the hydrostatic assumption and consider using non-hydrostatic models in the near future. Recently, Daley [8] suggested that non-hydrostatic processes have to be included in global models in the near future with the steady increase of model resolution. One way to include non-hydrostatic processes while eliminating the acoustic modes is to use anelastic approximations to the governing equations.

3.1.2. Anelastic (sound-proof) approximations. To study the behavior of convective phenomena in the atmosphere, Ogura and Philips [35] proposed the anelastic approximation and proved its validity through a rigorous scale analysis. The following discussion is based on the work of Ogura and Philips [35]. They separated the atmospheric variables into two parts, the base state and the perturbations from it. Two basic assumptions have been made in their study.

(1) The base state potential temperature is chosen as a constant; the potential temperature departure from the base state is small.
(2) The time scale of the motion is chosen to be the inverse of the well known Brunt-Vaisala frequency, N, which separates the gravity waves and acoustic waves in a resting isothermal atmosphere.

The first assumption implied that the variation of density from the horizontal mean is sufficiently small and can be ignored except for the density variations are multiplied by the term g (buoyancy term). Under the anelastic assumptions, the continuity equation can be written as

$$\frac{\partial u\rho(z)}{\partial x} + \frac{\partial v\rho(z)}{\partial y} + \frac{\partial w\rho(z)}{\partial z} = 0 \tag{3.1}$$

where ρ, the density of the base atmosphere is a function of Z only. A special case of the anelastic approximations is the incompressible Boussinesq approximations, in which the variability of density is assumed to be small and can be ignored everywhere except in the buoyancy term in the vertical momentum equation. Under incompressible Boussinesq assumption the continuity equation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{3.2}$$

The incompressible Boussinesq approximation is commonly used in modeling shallow convections in the atmosphere. Since the anelastic approximation eliminates the prognostic nature of the continuity equation, one must then solve an elliptic equation derived from momentum equation and continuity equation.

The anelastic system proposed by Ogura and Philips contains the following equations in addition to equation (3.1). They are

$$\beta\frac{d\mathbf{V}_0}{dt} = -\nabla\pi_1 \tag{3.3}$$

$$\beta\frac{dw_0}{dt} = -\frac{\partial\pi_1}{\partial z} + \beta\theta_1 \tag{3.4}$$

$$\frac{d\theta_1}{dt} = 0 \tag{3.5}$$

where all the variables are nondimensional, the base state variables have a subscript "0", and perturbations of the variables from the base state have a subscript "1". One important note concerning the anelastic system is that the variables v_0, w_0, π_1, and θ_1 are not completely independent. The nondimensional pressure π_1 must always be such that the forcing terms in momentum equations continue to satisfy the continuity equation. These constraints implied that π_1 must be determined by the solution of an elliptic equation:

$$\nabla \cdot \rho_0 \nabla \pi_1 + \frac{\partial}{\partial z}(\rho_0 \frac{\partial \pi_1}{\partial z}) = \beta \frac{\partial \rho_0 \theta_1}{\partial z} - \beta \nabla \cdot [\rho_0 (\mathbf{V_0} \cdot \nabla \mathbf{V_0} + w_0 \frac{\partial \mathbf{V_0}}{\partial z})]$$

$$-\beta \frac{\partial}{\partial z}[\rho_0 (\mathbf{V_0} \cdot \nabla w_0 + w_0 \frac{\partial w_0}{\partial z})]. \qquad (3.6)$$

Equations (3.3) to (3.6) do not allow acoustic waves as solutions. For modeling the deep dry convection, where mixing will keep the environmental lapse rate close to adiabatic, the anelastic assumption can be used with complete confidence. It will be adequate to use the incompressible Boussinesq approximation for modeling the shallow convections. if the purpose of the study is for deep moist convections, however, the departure of the potential temperature from the isentropic base state can be rather large ([9]). Several modifications to the anelastic approximations have been revised (e.g. Wilhelmson and Ogura [45]; Lipps and Hemler [28]; Durran [9]; Lipps [27]). Lipps and Hemler [28] suggested that the moist deep convections can be handled if the base state potential temperature is a slow varying function of height.

Durran [9] has suggested that the acoustic waves can be filtered from the model atmosphere with minimal changes to the governing equations by replacing the fully compressible continuity equation with the "pseudo-incompressible equation"

$$\nabla \cdot (\bar{\rho}\bar{\theta}\mathbf{V}) = \frac{H}{C_p \bar{\pi}}$$

where $\bar{\rho}(z)$ and $\bar{\theta}(z)$ are the vertically varying base state density and potential temperature, $\bar{\Pi}(z) = (\bar{p}(z)/p_0)^{R/C_p}$ is the base state Exner function, and H is the rate of heating per unit volume.

3.2. Numerical techniques

As discussed above, the fully compressible system admits the high frequency acoustic waves as solutions. The presence of the modes in the system will place severe limit on the time step for numerical integration. However, one can reduce computer time requirements by using either semi-implicit (e.g. Tapp and White [43]) or split-explicit (e.g. Klemp and Wilhelmson [24]; Gadd [13]) techniques. These integration techniques are described briefly in the following section.

3.2.1. Semi-implicit technique. Semi-implicit scheme was developed in the late 60's [38], to allow larger time steps than it would be possible using a conventional explicit time integration in a primitive equation model. Longer time steps are achieved by a substantial slowing of fast moving gravity waves. In the middle of the 70's, Tapp and White [43] adopted a semi-implicit scheme, similar to that described by Kwizak and Robert [22], for their fully compressible non-hydrostatic model. The idea behind this technique is to treat the dominant terms that govern the acoustic modes in the equation of motion implicitly. To illustrate how the semi-implicit technique used in practice, we briefly summarize the procedures that Tapp and White [43] used in their model.

They separated the Exner function P and potential temperature θ into a stable base state P_0 and a constant θ_0 and deviations P_1 and θ_1 as follows

$$P = P_0(z) + P_1(x, y, z, t)$$

$$\theta = \theta_0 + \theta_1(x, y, z, t)$$

where, $P_0 = 1 - z/H$ and $H = C_p\theta_0/g$ is a scale height for the basic atmosphere. Using these variables, equations (2.5)–(2.8) can be written as,

$$\partial u \partial t + C_p\theta_0 \frac{\partial P_1}{\partial x} = -\mathbf{V} \cdot \nabla u + fv - C_p\theta_1 \frac{\partial P_1}{\partial x} + F_1 \tag{3.7}$$

$$\frac{\partial v}{\partial t} + C_p\theta_0 \frac{\partial P_1}{\partial y} = -\mathbf{V} \cdot \nabla v - fu - C_p\theta_1 \frac{\partial P_1}{\partial y} + F_2 \tag{3.8}$$

$$\frac{\partial w}{\partial t} + C_p\theta_p \frac{\partial P_1}{\partial z} = -\mathbf{V} \cdot \nabla w + g\frac{\theta_1}{\theta_0} - C_p\theta_1 \frac{\partial P_1}{\partial z} + F_3 \tag{3.9}$$

$$C_p\theta_0 \frac{\partial P_1}{\partial t} - gw + C_0^2 \nabla \cdot \mathbf{V} = C_p\theta_0 [V \cdot \nabla P_1 + (\gamma - 1) P_1 \nabla \cdot V - \frac{(\gamma - 1)Q}{C_p(P_0 + P_1)}] \tag{3.10}$$

$$\frac{\partial \theta_1}{\partial t} = -\mathbf{V} \cdot \nabla\theta_1 + \frac{Q}{C_p(P_0 + P_1)} \tag{3.11}$$

where all the dominant terms that govern acoustic modes appear on the left-hand side and all the other terms on the right-hand sides. Here $C_0(z) = \sqrt{\gamma R \theta_0 P_0}$ is the phase speed of the acoustic waves in the base state atmosphere.

In finite difference form, all the terms on the left-hand side are treated implicitly as an average between time levels $n+1$ and $n-1$. For example the pressure gradient term $C_p\theta_0 \nabla P_1$ is represented by $C_p\theta_0 \nabla \frac{1}{2}(P_1^{n+1} + P_1^{n-1})$ and the acoustically active terms $-gw + C_0^2 \nabla \cdot V$ in continuity equation are replaced with $-g\frac{1}{2}(w^{n+1} + w^{n-1}) + C_0^2 \nabla \cdot \frac{1}{2}(V^{n+1} + V^{n-1})$. The terms on the right-hand side are evaluated either at time level n or $n-1$ depending on the stability requirements. After some algebra, the finite difference form of the equations (3.7) to (3.11) can be written as,

$$u^{n+1} = u^{n-1} + X - \delta t \frac{\partial \pi}{\partial x}$$

$$v^{n+1} = v^{n-1} + Y - \delta t \frac{\partial \pi}{\partial y}$$

$$w^{n+1} = w^{n-1} + Z - \delta t \frac{\partial \pi}{\partial z}$$

$$C_p\theta_0 P_1^{n+1} = P_1^{n-1} + C_p\theta_0\Phi - \delta t[g\Delta^2 w - C_0^2 \nabla \cdot \Delta^2 \mathbf{V}]$$

$$\theta_1^{n+1} = \theta_1^{n-1} + \Theta$$

$$\Delta^2 \mathbf{V} = \mathbf{V}^{n+1} - 2\mathbf{V}^n + \mathbf{V}^{n-1}$$

$$\pi = C_p\theta_0\Delta^2 P_1 = C_p\theta_0[P_1^{n+1} - 2P_1^n + P_1^{n-1}].$$

The terms X, Y, Z, Φ and Θ are $2\delta t$ times the right-hand side of the equations (3.7) to (3.11). Eliminating u^{n+1}, v^{n+1} and w^{n+1}, they obtained a three-dimensional elliptic equation:

$$\nabla_H^2 \pi + [\frac{\partial^2}{\partial z^2} - \frac{g\partial}{C_0^2 \partial z} - \frac{1}{C_0^2 \delta t^2}]\pi = F \tag{3.12}$$

where ∇_H^2 is the horizontal Laplace operator and F is a known function of variables at the present and past time levels only. If the boundary condition are known, equation (3.12) can be solved by either using direct or iterative solvers.

It has been shown that the computational stability of semi-implicit scheme is very sensitive to the base state temperature profile (e.g. Simmon et al. [40]; Cullen [5]; Tanguay et al. [42]). If an isentropic base state is chosen, one can only treat the terms governing acoustic waves implicitly (Tapp and White [43]). While as shown by Cullen [5] and Tanguay et al. [42] that the terms governing both the acoustic modes and the fast moving gravity modes can be treated implicitly if an isothermal base state atmosphere is used.

Longer time steps are achieved at the expense of substantial slowing of the acoustic modes and in some cases the fast moving gravity modes (e.g. Simmon et al. [40]). It will be necessary to keep the gravity modes explicit if those are meteorologically important, such as the scale interaction studies involving the gravity modes.

3.2.2. Split-explicit techniques. Similar to the idea of semi-implicit technique, the split-explicit technique includes two main steps.

(1) Separate the forcing terms into those that are dominant for acoustic modes and deviations from those terms,
(2) Smaller time steps are used for terms that govern acoustic waves, while larger time steps are used for the other terms.

To be practical, this scheme requires that only a few terms in the governing equations by important for acoustical waves. Following Cotto and Tripoli [4] the split-explicit technique is described as follows. The momentum equations can be written as,

$$\frac{\partial \bar{U}_i}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i} + RU_i \tag{3.13}$$

$$\frac{\partial p'}{\partial t} = \gamma p_0 \frac{\partial \bar{U}_j}{\partial x_j} + RP \tag{3.14}$$

where RU_i and RP are the terms governing acoustic waves for the base atmosphere and $(1/\rho_0)\partial P'/\partial x_i$ and $\gamma P_0 \partial \bar{U}_j/\partial x_i$ are the deviations. A large time step Δt_L determined from meteorologically phase speeds with leapfrog scheme is used to approximate time tendency terms, while a smaller time step Δt_s is used for the left side terms. They let $\Delta t_L = N\Delta t_s$, where N is an integer. The right side terms, RU_i and RP, are evaluated at the $\tau - n\Delta t_s$ time level for the diffusion terms and τ level for advection and other terms. The left side terms are evaluated on a time step Δt_s in a marching process between time level $\tau - N\Delta t_s$ and $\tau + N\Delta t_s$ in time steps of $2\Delta t_s$. This process appears in

$$\bar{U}_i^{[\tau - N\Delta t_s]+2n\Delta t_s} = \bar{U}_i^{[\tau - N\Delta t_s]+2(n-1)\Delta t_s} + 2\Delta t_s \times \left[\left(\frac{1\partial P'}{\rho_0 \partial x_i} \right)^{[\tau - N\Delta t_s]+2(n-1)\Delta t_s} + RU_i^{\tau, \tau - N\Delta t_s} \right], \tag{3.15}$$

$$p'^{[\tau - N\Delta t_s]+2n\Delta t_s} = p'^{[\tau - N\Delta t_s]+2(n-1)\Delta t_s} + 2\Delta t_s \times \left[\left(\gamma P_0 \frac{\partial \bar{U}_j}{\partial x_j} \right)^{[\tau - N\Delta t_s]+2(n-1)\Delta t_s} + RP_i^{\tau, \tau - N\Delta t_s} \right], \tag{3.16}$$

where $n = 1, \ldots, N$ is the small time step iteration level.

3.3. Boundary conditions

Unrealistic lateral boundary conditions are believed to be a major source of error in regional primitive equation models because of the ill-posed mathematical nature of the problem [1]. Same is true for the non-hydrostatic models. This section describes briefly a few methods to provide boundary condition and to reduce or eliminate reflection of fast moving waves at boundaries.

One popular method is the sponge (absorbing) method, proposed by Perkey and Kreitzberg [37]. The sponge method involves utilizing an increased horizontal eddy viscosity in a narrow band around the lateral boundary. The purpose of this method is to minimize spurious reflection of wave at the boundaries damping them in the sponge region. To allow the information obtained from a large scale model to propagating into the domain, a merging term may be added to the prognostic equations in the boundary zone. Durran and Klemp [10] applied a sponge layer at the upper boundary of their non-hydrostatic model to damp spurious energy reflection from the upper boundary.

The other common method is radiation lateral boundary condition, which minimizes the spurious reflection of an outward propagating wave. The radiation boundary conditions at lateral boundaries, suggested by Orlanski [36], are commonly used in which the phase speed of a gravity wave impinging on the boundary is estimated, and all of the variable are advected out of the boundary at the speed. The success of this method depends on choosing correct phase speed. There are some modifications of the original Orlanski scheme suggested by several authors (e.g. Klemp and Lilly [23]; Klemp and Wilhelmson [24]). Miyakoda and Rosati [34] have found the radiation lateral boundary conditions to be superior to the sponge method.

It has been found that a properly provided upper boundary condition is very crucial for a successful simulation when the vertically propagating large amplitude waves, such as mountain waves ([10], [25]), are present. A radiation upper boundary condition requires that all energy transported upwards be radiated out through the upper boundary. Durran and Klemp [10] reported that the radiation upper boundary condition proposed by Klemp and Durran [25] to be superior to the sponge layer at the upper boundary.

4. Physical processes

The real atmosphere contains a variety of physical processes with different characteristic length scales. Some of the processes may not be meteorologically important, while others are essential for weather systems. Inclusion of these processes in a numerical model depends on the model resolution. When the model resolution is not fine enough the implicit methods (or parameterizations) are commonly used to represent subgrid scale physical processes. For fine resolution models, some of the processes can be included explicitly, while some of the processes, such as turbulence, may still have to be parameterized. For example, because the non-hydrostatic processes are generally not important in synoptic scale motions, one can simply employ the same physical packages used in hydrostatic large scale models even though a fully compressible non-hydrostatic system is used. For very fine model resolutions, where non-hydrostatic processes becomes very important, some physical processes may need to be treated explicitly, such as convective clouds (e.g. Yamasaki [47]; Lord et al. [30]; Rotunno and Emanuel [39]). Most of the non-hydrostatic models are designed to study or forecast the small scale motions, where some modifications on the commonly used parameterization schemes, such as Kuo scheme, in large scale prediction models are needed. Inclusion of physical processes in the hydrostatic and non-hydrostatic models are quite similar. To give an example of how the physical processes are included in non-hydrostatic operational forecast model, the following section briefly summarizes the representation of physics in the non-hydrostatic version of the British Meteorology Office's weather forecast model ([2], [15]).

Boundary layer processes are the important processes that have a large impact on the weather systems. The radiation and turbulent diffusion processes are the two major components of the surface heat budget in the model. Both processes are controlled by the characteristics of the ground, such as surface roughness, wetness, albedo, conductivity, and vegetation. The surface processes over land are treated differently from the ones over ocean. The diurnal changes of surface processes and the presentation of cloud are also incorporated. The Monin-Obukhov similarity theory is used to calculate the mixing coefficients of the turbulent diffusion processes between the ground surface and first level (at 10 m); the mixing coefficients above the first level are calculated through the TKE (turbulent kinetic energy) closure, and the mixing length. The local change of TKE is determined from advection, turbulent transport, shear production, buoyancy, and dissipation, while the mixing length is diagnosed and assumed to increase above the ground. The boundary layer cloud is also predicted in the model.

The nonconvective clouds are one of the main sources that produce precipitation. They first calculate the subgrid scale relative humidity accounting the turbulent variations of the conservative variables and include a term that depends on layer depth to model non-turbulent variations. The amount of saturated air is calculated from mean humidity and its variance by assuming a probability distribution. The treatment for convective clouds is somehow different from the one in large scale model. To parameterize the very important convective processes, the parameterization scheme by Fritsch and Chappell [12] is used in the model. The major difference between this scheme and the ones used in large scale model is that the cloud has a specified lifetime and the cloud can move during its life. The details of the cloud life cycle are not included.

5. Initialization

Because the numerical weather forecast is basically an initial value problem, initialization would be the most important part leading to a successful numerical weather forecast if a perfect model were available. The observations we get generally include different scales of motions and physical processes as well as observational errors. To get a set of dynamically and thermodynamically consistent initial data initialization procedures are commonly employed. Because of the availability of more and more satellite and other asynoptic data, a process of merging new observational data with the ongoing integration of a numerical forecasting model known as "four-dimensional data assimilation" is increasingly used. Excellent review papers on this subject are provided by Daley [7] and Harms et al. [17]. Here, only a brief summary of some of the schemes used in non-hydrostatic models is given.

There are several different initialization techniques, such as static initialization, dynamic initialization, normal mode initialization, nonlinear normal mode initialization, vertical mode initialization, inverse Laplace transformation, and bounded derivative method. The most commonly used scheme in numerical weather forecasting is nonlinear normal mode initialization scheme. For most non-hydrostatic models used for research, the initial field is commonly generated from a single sounding, because of lack of observation data in the small model domain. In the British Meteorology Office's non-hydrostatic model the initial model input data is combined from three sources: (1) an interpolation of the latest fine mesh forecast (from a 6 hour forecast), (2) a 3 hour mesoscale forecast, and (3) the surface synoptic observations.

Although there are various four-dimensional data assimilation methods, such as intermittent data assimilation (or analysis-forecast cycle), Kalman-Bucy filtering, and the adjoint method using variational techniques for hydrostatic models, few are used in preparing data for a non-hydrostatic model. Recently, Kapitza [20] adopted the adjoint method for an anelastic non-hydrostatic model [21] to merge the asynoptic radar data into the model.

6. Summary

In this paper, an attempt has been made to offer a brief review of non-hydrostatic numerical models of the atmosphere for both operational and research purposes. We described fully compressible non-hydrostatic equations and various vertical coordinate systems used in non-hydrostatic models. We have also discussed various assumptions, such as hydrostatic assumption and anelastic approximations, used to eliminate the fast propagating acoustic waves in the model atmosphere. Two commonly used time integration techniques for the fully compressible non-hydrostatic system are briefly reviewed. Finally, the inclusion of physical processes and initialization and four-dimensional data assimilation were presented.

Recently modeling studies (e.g. Cullen [5]; Tanguay et al. [42]) have shown that one can use a fully compressible non-hydrostatic model to simulate various weather systems without any computational penalty compared to a similar hydrostatic model. More studies on various ways to include physical processes effectively, such as implicit, or explicit approaches, or a combination of the two, are needed. We need to investigate the effect of nesting a fine mesh non-hydrostatic model within a coarser non-hydrostatic model. Much research is needed in merging the asynoptic data, such as data from radar, lidar, and profiler, in predicting the local severe weather.

Acknowledgments. This work was supported by the Naval Research Laboratory, Washington, D.C.

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