VALIDITY OF THE LOG-LINEAR PROFILE RELATIONSHIP OVER A ROUGH TERRAIN DURING STABLE CONDITIONS*

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Abstract. The applicability of the log-linear profile relationship over rough terrain to a height of 126 m is investigated. Simultaneous hourly averaged mean wind and temperature profiles measured at the Brookhaven meteorological tower during stable conditions are used in the analysis. The tower was surrounded by fairly homogeneous vegetation to a height of about 8 m. The results indicate that the log-linear profile relationship is valid at least for a height of 126 m for stabilities with Richardson numbers less than the critical value of 0.25. The mean value of α in $\partial u/\partial z = (u_*/k)\,(1 + \alpha(z/L))$ is found to be about 5.2 for these stabilities. The log-linear profile relation is found to be applicable for profiles observed beyond the critical stability; but the height of validity seems to decrease to about 100 m and the mean value of α is about 1.6.

1. Introduction

Variation of the mean wind speed with height during near neutral stability conditions is fairly well understood and documented with experimental data. A logarithmic relationship has been found to satisfy the observations in the atmospheric surface layer extending to about 100 m. However, when the conditions are appreciably different from neutral, as in the case of inversions and superadiabatic lapse rates, profile relationships are not well understood. For the neutral stability conditions, the Richardson number and the similarity parameter z/L, where z is the height above the surface and L is the Monin Obukhov length, assume a unique value of zero. For inversion or superadiabatic conditions, these parameters have varying positive or negative values, respectively. Hence, it is natural to expect that any profile relationship for stability conditions other than neutral would depend on the degree of stability. The practice has been to adopt constant values for α for stable and unstable conditions in the truncated power-series relationship suggested by Monin and Obukhov of the form

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}\left[1 + \alpha\left(\frac{z}{L}\right)\right] \tag{1}$$

where u is the mean wind speed, z the height, u_* the friction velocity, L the Monin–Obukhov length and k von Karman's constant (~ 0.4). This expression reduces to the familiar logarithmic form for neutral conditions. Several values varying from 4.5 to 7 have been suggested for α for stable atmospheric conditions (Lumley and Panofsky, 1964). More recently, Webb (1970) suggested a value of 5.2, Businger *et al.* (1971) 4.5 to 5.0 and Carl *et al.* (1973) a value of 5.0.

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Different values obtained by different investigators suggest the necessity of further investigation. Moreover, the above results, except for those of Carl *et al.* (1973), were based on observations within a height of 16 m over flat, homogeneous terrain. Tower observations reported by Carl *et al.* (1973), were over generally flat terrain. An attempt is made in this paper to determine the log-linear profile relationships for the flow over an extremely rough but relatively homogeneous terrain. The maximum height to which the observations were made was 126 m. The atmospheric stabilities analyzed varied from slightly stable to very strong inversion conditions.

2. Observations

2.1. Site

The mean wind and temperature measurements were made on the 126-m meteorological tower at Brookhaven National Laboratory. In the immediate vicinity of the tower, the terrain is fairly flat. Beyond a 500-m radius, the vegetation consists mainly of scrub pine and oak about 8 m in height on average, as shown in Figure 1. This fairly homogeneous vegetation exists for several kilometers in all directions except for a 30-deg arc east-south-east of the tower which contains low-frame buildings at a distance of 0.5 to 1 km.

2.2. Measurements

Hourly averaged mean wind speeds were measured with sensitive three-cup anemometers at heights of 5.5, 11.5, 23, 46, 109 and 126 m. Mean temperatures were measured by aspirated Leeds and Northrup resistance sensors at heights of 11.5, 23, 46, 92 and 126 m. An additional mean temperature measurement was available at a height of 2 m nearby. The anemometers and the temperature sensors were periodically calibrated in a subsonic wind tunnel and an environmental chamber, respectively. The data analyzed here are for April–June, 1965.

3. Log-Linear Wind Profile Analysis

Log-linear profiles expressed by Equation (1) in gradient form can be integrated to yield mean wind speed u at level z as

$$u=\frac{u_*}{k}\left\{\ln\frac{z}{z_0}+\alpha\left(\frac{z-z_0}{L}\right)\right\} \tag{2}$$

where z_0 is the roughness length and L is Monin–Obukhov length defined by

$$L = -u_*^3 c_p \rho\theta(kgH)^{-1} \tag{3}$$

where H is the vertical heat flux assumed positive upwards, c_p is the specific heat of air at constant pressure, ρ is the air density, g is the gravitational acceleration and θ is the potential temperature.

Fig. 1. Meteorological tower at Brookhaven National Laboratory.

Following the approach adopted by Webb (1970), integration of Equation 2 between two heights z_1 and z_2 yields

$$\frac{u_2-u_1}{\ln(z_2/z_1)}=\frac{u_*}{k}\left\{1+\frac{\alpha}{L}\frac{z_2-z_1}{\ln(z_2/z_1)}\right\} \tag{4}$$

where u_1 and u_2 are the mean wind speeds at heights z_1 and z_2, respectively. With a zero-plane displacement δ often found over rough terrain, Equation (4) could be written as

$$y=\frac{u_*}{k}\left\{1+\frac{\alpha x}{L}\right\} \tag{5}$$

where the variables y and x are given by,

$$y=\frac{u_2-u_1}{\ln\dfrac{z_2-\delta}{z_1-\delta}} \quad \text{and} \quad x=\frac{z_2-z_1}{\ln\dfrac{z_2-\delta}{z_1-\delta}}. \tag{6}$$

An advantage of this method is that the roughness length z_0 can be eliminated and the only unknowns are α and L. The observations to be analyzed here are simultaneous mean wind and mean temperature profiles. Due to the unavailability of heat flux measurements, L had to be estimated from the Richardson number, Ri, using the definition of Richardson number and the log-linear relationships for mean wind and temperature gradients (Webb, 1970). This yields,

$$\mathrm{Ri}_z = \frac{z}{L}\left\{1+\alpha\frac{z}{L}\right\}^{-1}. \tag{7}$$

Intercept on x-axis, x_0 in Equation (5) is given by,

$$x_0 = -L/\alpha. \tag{8}$$

Combining Equations (7) and (8),

$$\alpha = z\{\mathrm{Ri}_z(z-x_0)\}^{-1}. \tag{9}$$

For some of the observations, measurements of mean temperature were available at sufficient number of levels to compute the heat flux $H = C_p\rho u_* T_*$ from the relation

$$\frac{\partial\theta}{\partial z} = \frac{-T_*}{kz}\left\{1+\alpha\frac{z}{L}\right\} \tag{10}$$

based on the assumption that the profiles of $\partial u/\partial z$ and $\partial\theta/\partial z$ are similar. The values of L obtained by this method were compared with the one obtained from the Richardson number and were found to be within a few percent of each other.

4. Discussion of Results

4.1. Characterization of atmospheric stability

Atmospheric stability for the cases studied has been characterized by the Richardson number at the geometric mean height of 23 m in finite-difference form, defined as

$$\mathrm{Ri}_{23} = \frac{g}{\bar{\theta}}\sqrt{z_1 z_2}\,\ln\frac{z_2}{z_1}\,\frac{(\theta_2-\theta_1)}{(u_2-u_1)^2} \tag{11}$$

where g is the gravitational acceleration, θ_1 and θ_2 are the absolute potential temperature at heights 11.5 and 46 m, respectively, $\bar{\theta}$ is the mean absolute potential temperature in the layer considered, $z_1 = 11.5$ m and $z_2 = 46$ m.

Another Richardson number (Ri_{38}) based on heights of 11.5 and 125.8 m was computed to determine the degree of variability. Although the variation between Ri_{23} and Ri_{38} was 10 to 40%, α's computed from them independently did not vary by more than 10% for the corresponding Richardson numbers. Ri_{23} for the observations used here varied from 0.09 to 1.7 corresponding to the range from slightly stable conditions to very strong inversions. Most of the observations were within a Ri_{23} of 0.7.

4.2. Similarity of Wind and Temperature Profiles

One of the basic assumptions in the formulation of Equation (7) is that mean wind and temperature profiles are similar. This requirement is essential to compute reliable values of α from profile measurements. Turbulent exchange coefficients K_M and K_H are defined by

$$K_M = \frac{\tau}{\rho \partial u/\partial z}, \qquad K_H = \frac{-H}{c_p \rho \partial \theta/\partial z} \tag{13}$$

where K_M and K_H are the exchange coefficients for momentum and heat, respectively, τ is the horizontal shearing stress, H is the vertical heat flux, θ is the potential temperature, ρ is the air density and c_p is the coefficient of heat at constant pressure. Friction velocity u_* is given by $(\tau/\rho)^{1/2}$. The profiles are similar when the ratio of the vertical gradient of potential temperature is constant with height. An ideal way of determining K_H and K_M would be from simultaneous gradient and flux measurements. In the absence of such measurements, some information can still be obtained from mean wind and temperature gradients.

A parameter of profile similarity commonly used is defined as

$$P = (\Delta u/\Delta \theta)_2/(\Delta u/\Delta \theta)_1 \tag{14}$$

where the subscripts 1 and 2 pertain to the lower and upper level intervals, respectively, through which hourly averaged mean wind and temperature differences are computed. The parameter P would have a value of one if the profiles are similar or K_H/K_M is constant in the height considered. If P is less than one, K_H/K_M decreases with height and a value greater than one indicates an increase of the ratio of exchange coefficients with height.

Hourly mean winds and temperatures at 11.5, 23, and 46 m were used to compute the parameter P. The mean wind speeds measured at 5.5, 11.5, 23, 46, 108.9, and 126 m were used to fit the log-linear relationship given by Equation (5). A similar log-linear fit was made for the mean temperatures measured simultaneously at 11.5, 23, 46, 92 and 125.8 m. The profile parameter was computed for cases when the regression coefficients for the least-square fits of Equation (5) for the wind profile R and temperature profile R_u were each more than 0.99, indicating a good fit. There were 163 wind profiles in the 69 days of continuous observations that passed this criterion. Of these 163 events, only 122 satisfied the criteria that the regression coefficient be greater than 0.99 for both the wind and temperature profiles.

The profile parameter, P, was evaluated for each of the 122 cases mentioned above. The mean values are listed in Table I for different ranges of Richardson number. Number of observations in each range and the standard deviation of P are also given.

Analysis of the profile parameter, as shown in Table I, indicates near constant mean values up to a Ri_{23} of 0.25, an increase at 0.25 and a decrease thereafter. The standard deviation of P tends to increase with the Ri_{23}.

TABLE I
Values of profile parameter for different atmospheric stabilities

Range of Ri_{23}	Mean P	Std. deviation of P	Number of Observations
0.05–0.10	1.23	0.27	33
0.10–0.15	1.24	0.25	40
0.15–0.20	1.26	0.20	8
0.20–0.25	1.38	0.32	10
>0.25	0.96	0.41	31

There does not seem to be any significant variation of P with stability and the values are in general close to one. Analysis by Webb (1970) for P of wind profiles measured over flat terrains up to heights of 16 m also indicates no substantial variation of P with stability. He obtained values of P somewhat larger than the one for stable cases. It is interesting to note that the non-dimensional profile parameter, P and hence, K_H/K_M behave in the same way over both flat and rough terrains.

Data were then divided into two sets: (1) observations with $0.9 < P < 1.1$ and with R_u and R_θ greater than 0.99; and, (2) all observations with $R_u = R_\theta > 0.99$. The purpose of set 1 was to apply rigorous conditions before computing α so that the complexity introduced by rough terrain could be avoided to a certain extent. Moreover, if there is any systematic difference in the value of α due to P being different from one, it would be visible from a comparison of results of data sets 1 and 2. There were 24 observations that satisfied the requirement, $0.9 < P < 1.1$ as shown in Figure 2 for Ri_{23} varying from 0.10 to 0.69. It seems that the ratio tends to be near one for small Ri's. Businger *et al.* (1971) found K_H/K_M to be about 1.35 in neutral conditions, decreasing slowly with increasing stability.

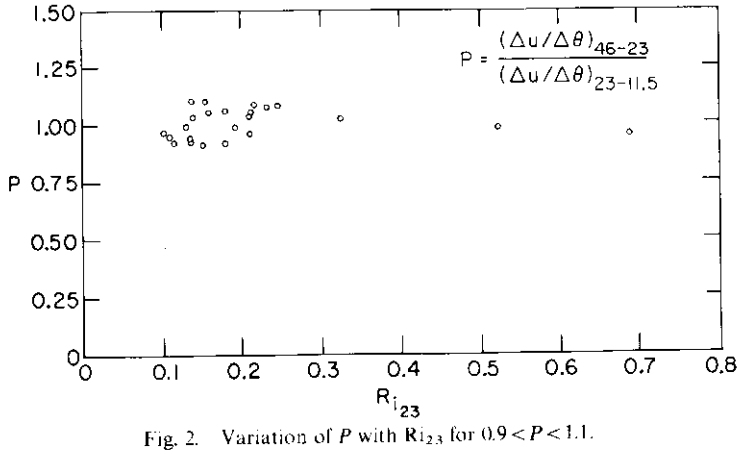

Fig. 2. Variation of P with Ri_{23} for $0.9 < P < 1.1$.

4.3. Determination of α with mean wind profiles for $0.9 < P < 1.1$

Hourly averaged mean wind measurements at six levels were used to compute the x and y coordinates (Equation (6)) in the modified log-linear relationship given by Equation (5). The trees surrounding the meteorological tower were essentially uniform with an average height of about 8 m. Displacement length, δ was assumed to be 4 m, half the height of trees. When the log-linear regression coefficient, R_u, was greater than 0.99, a change in δ by 2 m did not produce any significant change in R_u and in the location of intercepts.

Figures 3 and 4 show typical simultaneous, hourly mean wind and temperature profiles during inversion conditions with a Ri_{23} of 0.21. While plotting Equation (5) with the x and y coordinates given by Equation (6), two sets of coordinates are available. They are adjacent pairs of heights, z_2 being the next higher level of measurement after z_1, and other pairs of heights. This increases the number of sets of points for any particular profile although the latter is not completely independent of the former. Since there were six levels at which mean wind speeds were measured, five adjacent pairs of heights were available, sufficiently large to give confidence in the profiles.

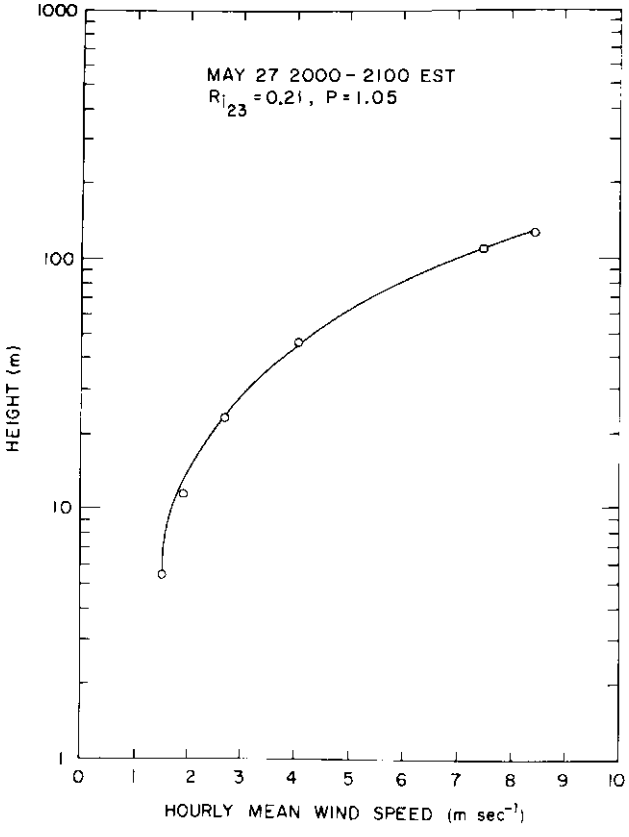

Fig. 3. A typical hourly mean wind profile during inversion conditions.

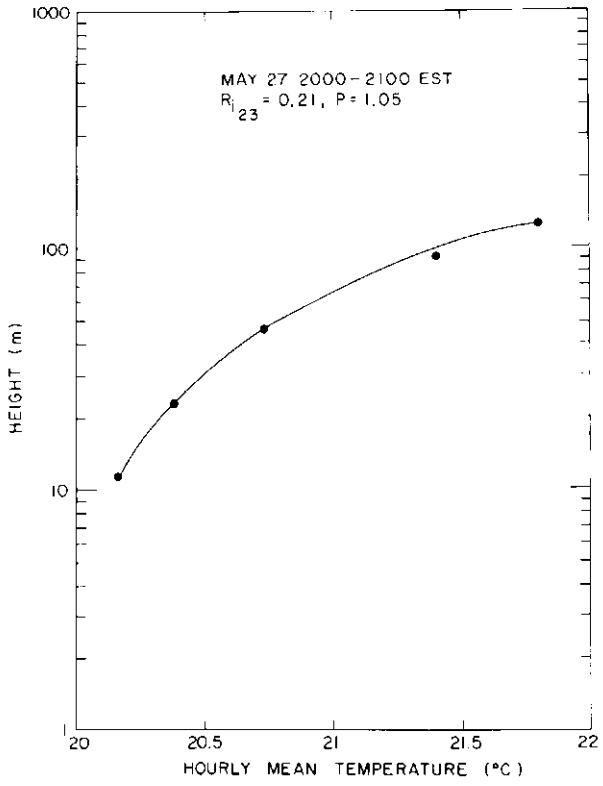

Fig. 4. Simultaneous hourly mean temperature profile for Figure 3.

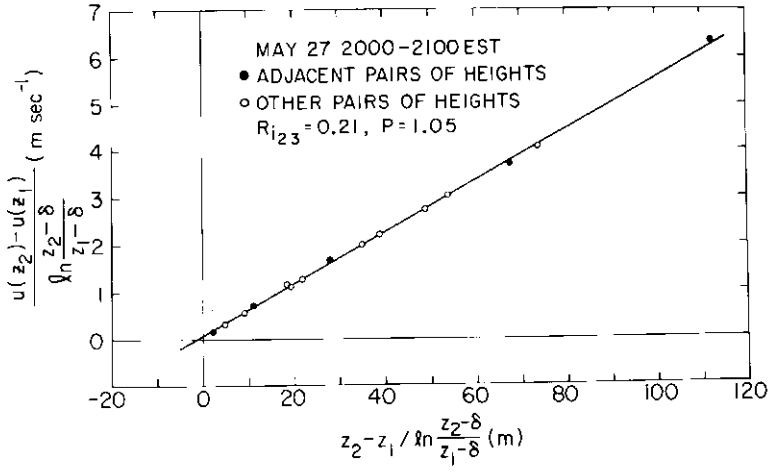

Fig. 5. Log-linear plot for the wind profile of Figure 3.

The mean wind profile of Figure 3 is plotted in log-linear form in Figure 5. The regression coefficient for the least-square fit was 0.999 indicating a very good fit. The Ri_{23} for this hour was 0.21. The mean temperature profile for the same period in log-linear coordinates is shown in Figure 6. Here also the regression coefficient was in excess of 0.99. Both the wind and temperature profiles have the same intercept x_0 in the horizontal axis. A value of 6 for α was obtained from these profiles, using Equation (9). A similar analysis was done for all the hourly observations during stable conditions that satisfied the following three requirements simultaneously. $R_u > 0.99$, $R_\theta > 0.99$ and $0.9 < P < 1.1$. The arithmetic mean of P for these data was 1.00 (geometric mean of 1.06) with a standard deviation of 0.06. The results are given in Table II which indicate several features of interest:

(1) The log-linear profile relationship seems to be valid over rough terrain to a height of at least 100 m for atmospheric stabilities ranging from slightly stable to strong inversions.

(2) The value of α seems to vary with the degree of stability.

(3) The profile parameter P was close to one for Ri_{23} as high as 0.7 indicating the possibility of K_H/K_M being invariant with height for strong inversions.

Webb (1970) found that the profiles deviated from log-linear form for $z > L$ or $z/L > 1$. He attributed this deviation to the possible variability of shearing stress with height. Present data indicate that the profiles are log-linear for a Ri_{23} as high as 0.7 and, in fact, for Richardson numbers of about 1.0 as will be shown later in this paper. The reason for this variability of the limiting Ri or z/L is not known although the effect of the increased mechanical roughness cannot be ruled out.

One would expect K_H/K_M (or P) to decrease with height for strong stable conditions due to the increase of buoyancy forces. The results in Table II indicate P to be nearly one for Richardson numbers up to 0.8. This could be due to the strong correlation between the wind and temperature fluctuations that is known to exist and the possibility of the thermal buoyancy affecting the momentum and heat transfer to the same extent. Beyond a Ri_{23} of 0.7, P has a trend towards values less than one.

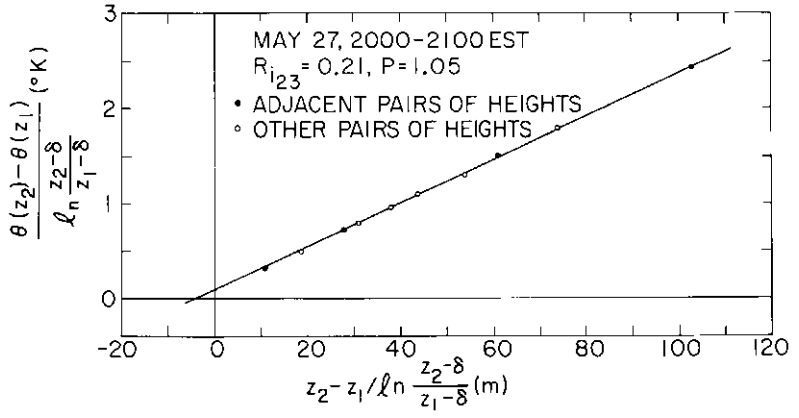

Fig. 6. Log-linear plot for the temperature profiles of Figure 4.

TABLE II
Values of α obtained from wind profiles with $0.9 < P < 1.1$

Day		Time (EST)	Wind direction	Ri_{23}	α
Apr.	9	0300–0400	66	0.24	3.25
	16	0700–0800	267	0.21	4.64
	21	2200–2300	177	0.19	3.78
May	4	0200–0300	255	0.15	6.33
	5	0000–0100	336	0.11	5.99
	8	2000–2100	198	0.32	2.43
	10	0500–0600	231	0.12	7.90
	11	0300–0400	261	0.18	4.74
	12	2000–2100	222	0.16	5.73
	16	0000–0100	240	0.18	4.50
	16	0200–0300	255	0.22	3.82
	17	2100–2200	312	0.14	5.46
	18	0400–0500	324	0.21	2.89
	21	2100–2200	150	0.52	1.68
	22	0200–0300	189	0.68	1.31
	27	1900–2000	255	0.21	4.29
Jun	6	0300–0400	234	0.14	5.81
	6	0400–0500	234	0.13	6.55
	6	0600–0700	234	0.23	3.71
	8	0500–0600	222	0.16	6.21
	9	0100–0200	225	0.14	6.89
	9	0400–0500	228	0.14	6.29
	9	0500–0600	228	0.14	6.28
	9	0600–0700	234	0.10	7.95

The results in Table II also show the variability of α with Richardson number. This was expected from Equation (9). Figure 7 shows the variation of α with Richardson number for set 1 ($0.9 < P < 1.1$). Values of α given by Webb (1970) are plotted in the figure for comparison, although his Richardson numbers were computed at the geometric mean height of 1.6 m. A systematic decrease in α with increasing Richardson number is apparent from Figure 7. The range of values obtained here agrees with that of Webb for the corresponding stabilities. Since Richardson number is known to increase with height, $Ri_{1.6}$ should be multiplied by a factor greater than one to obtain Ri_{23}.

Variations of α correspond to two distinct ranges of stabilities defined by Richardson number $Ri_{23} < 0.25$ and $Ri_{23} \geq 0.25$. The log-linear relationship was valid for both ranges, but there was a significant decrease in the value of α as the stability increased beyond $Ri_{23} = 0.25$. This value of Richardson number has often been found to be a critical value beyond which turbulence is severely damped by buoyancy forces. Webb (1970), McVehil (1968) and Businger *et al.* (1971) are some of the investigators who found the critical Richardson number to be close to 0.2. The present analysis shows that this value is not sharply defined but is roughly equal to 0.25.

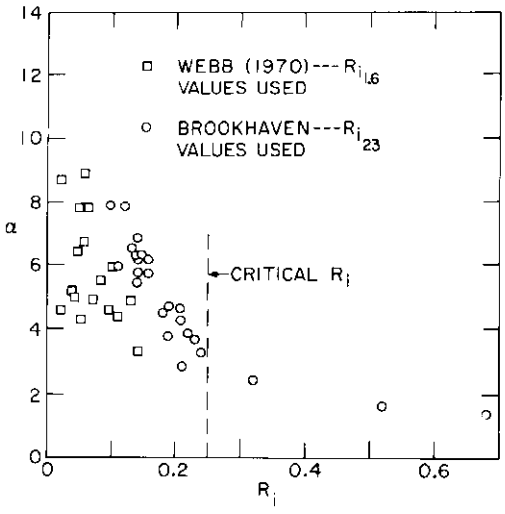

Fig. 7. Variation of α with Richardson number.

Of the 24 observations shown in Table II, 21 are with $Ri_{23} < 0.25$ and 3 are with $Ri_{23} \geq 0.25$. The geometric mean of the α values for $Ri_{23} \geq 0.25$ is 1.75 with a standard deviation of 0.57 and for $Ri_{23} < 0.25$ it is 5.2 with a standard deviation of 1.45. The number of observations with $Ri_{23} \geq 0.25$ is too small for meaningful statistical inference. But as we shall see in a later section, these α values are quite representative of the values to be obtained for all the profiles that satisfy the requirement of log-linearity with restrictions on the profile parameter P relaxed. McVehil (1964) has reported α to be around 7 for stable conditions with $Ri < 0.15$ and Webb (1970) found α to vary from 4.9 to 7.2 for the different sites ($Ri < 0.14$) that he analyzed. Standard deviations for individual α values for the observations reported by Webb (1970) varied from 8 to 32% depending on the site.

The mean α value of about 1.75 for $Ri_{23} > 0.25$ is not totally surprising due to the fact that x_0 approaches a value close to zero as the stability increases, resulting in a monotonic decrease of α with increasing Ri in Equation (9).

4.4. Mean wind and temperature profiles with strong inversions ($Ri_{23} \geq 0.25$)

Figure 8 shows a typical log-linear wind profile measured during a strong inversion condition with $Ri_{23} = 0.52$. The least-square regression coefficient was in excess of 0.99 for both mean wind and temperature profiles up to a height of about 100 m, beyond which the profiles deviated from log linearity. This feature was observed for all wind and temperature profiles with $Ri_{23} > 0.25$.

4.5. Values of α for all wind profiles with no restriction on the variability of K_H/K_M

As indicated in a previous section, log-linear analysis was carried out on all hourly mean wind profiles from data set 2 without restrictions on the variability of the profile

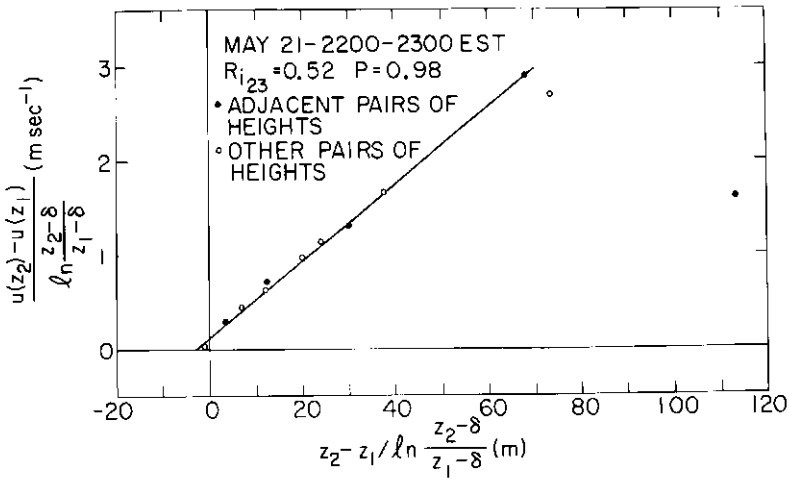

Fig. 8. Log-linear wind profile for a typical strong inversion condition.

parameter P (or K_H/K_M) with height. Values of P ranged from 0.48 to 1.82 and Ri_{23} generally varied from 0.08 to 1.57 except for one set of observations corresponding to a Ri_{23} of 2.69. The values of α computed from these profiles were similar to those found for profiles with $0.9 < P < 1.0$. The results are given in Table III. There were 98 profiles that met the requirement with regard to the log-linearity, i.e., R_u, $R_\theta > 0.99$ but did not have a profile parameter, P between 0.9 and 1.1. Of these, 70 had a $Ri_{23} < 0.25$ while the rest were above this critical Richardson number. As can be seen from Table III, the values of α obtained for these profiles are approximately the same as those with $P \simeq 1$.

The α values have a standard deviation of about 25% of the mean for each category. Webb (1970) found a similar scatter in his results. Mean values of 5.2 for $Ri < 0.25$ and 1.6 for $Ri > 0.25$ seem appropriate.

5. Conclusions

The analysis presented here shows that a log-linear profile relationship is valid for very strong inversion conditions ($Ri > 0.25$) for atmospheric flow over a rough terrain. This contradicts previous observations made over fairly flat, homogeneous terrain. This difference could be due to the effect of increased mechanical roughness in enhancing turbulence. A critical Richardson number seems to exist around 0.25, beyond which the profiles behave in a different manner although log-linearity still seems to be valid up to a height of 100 m. The difference in behavior is reflected in the reduction of α at Richardson numbers greater than 0.25. One would expect that at $z/L \gg 1$, the neglected second- and third-order terms in Equation 1 would become important and that the log-linear relationship would tend to break down. For the range of stabilities with $z/L > 1$ considered (up to a value of 6), the first-order log-linear relationship still seems to be valid. The quantity $\alpha(z/L)$ reaches an asymptotic, gradually increasing value with stability beyond $z/L = 1$.

TABLE III
Summary of α values obtained from wind profiles (with R_u and $R_\theta > 0.99$)

P	Ri_{23}	Number of hourly mean observations	Geometric mean α	Standard deviation of individual values	Mean values of α with 95% confidence interval
$0.9 < P < 1.1$	< 0.25	21	5.19	1.45	4.57 to 5.81
	≥ 0.25	3	1.75	0.57	1.18 to 2.32
$P < 0.9$ and > 1.1	< 0.25	70	5.73	1.54	5.37 to 6.09
	≥ 0.25	28	1.59	0.89	1.26 to 1.92
All profiles	< 0.25	91	5.41	1.53	5.10 to 5.72
	≥ 0.25	31	1.60	0.86	1.30 to 1.90

The mean value of 5.2 for α presently adopted is in good agreement with the results found in this analysis for Richardson numbers less than 0.25; but a variability of about 25% of this mean value should be taken into account when using this value. For wind profiles with Richardson numbers greater than 0.25 (the critical value), the height of validity decreases to 100 m and α seems to have a mean value around 1.6 with the same scatter, at least over a homogeneous, rough terrain. One of the reasons proposed in the past for the breakdown of log-linear relationship for $z/L > 1$ is the presence of gravity waves. The possibility of their presence for some of these observations with high Ri's could not be disputed, but the averaging of the wind for one hour and the presence of very rough terrain might have neutralized the effect of such irregularities in the flow during strong stable conditions.

The ratio K_H/K_M remained reasonably constant for a wide range of stable Richardson numbers and gradually decreased in value for very strong inversions. This is in agreement with the results of Webb (1970) and Businger *et al.* (1971) but in contradiction with the observations of Carl *et al.* (1973).

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