

Tropical Cyclone Simulations with the Betts Convective Adjustment Scheme. Part III: Comparisons with the Kuo Convective Parameterization

JONG-JIN BAIK

Department of Marine, Earth, and Atmospheric Sciences, North Carolina State University, Raleigh, North Carolina

MARK DEMARIA

Hurricane Research Division, AOML/NOAA, Miami, Florida

SETHU RAMAN

Department of Marine, Earth, and Atmospheric Sciences, North Carolina State University, Raleigh, North Carolina

(Manuscript received 2 March 1990, in final form 8 May 1991)

ABSTRACT

Numerical simulations of tropical cyclones in an axisymmetric model with the Betts convective adjustment scheme and the 1974 Kuo cumulus parameterization are compared. It is shown that the storm with the Betts scheme has a slightly more intense mature stage than the storm with the Kuo scheme. For both schemes, the parameterized heating is dominant initially, while the grid-scale heating is dominant at the mature stage. The storms begin to intensify rapidly when the grid-scale heating extends through a deep layer. The Betts scheme is more effective at removing water vapor and delays the onset of grid-scale heating. This results in later development of the storm with the Betts scheme. The storm evolution with both the Betts and Kuo schemes is sensitive to the treatment of the evaporation of liquid water in the grid-scale condensation scheme. This suggests that a prognostic equation for liquid water should be used when simulating tropical cyclones with a model resolution fine enough for grid-scale heating to be important.

1. Introduction

Since Kuo (1965, 1974) introduced a method for parameterizing deep cumulus convective processes, it (and its relatives) has been used widely in large-scale and mesoscale numerical models. The Kuo method is attractive relative to mass-flux-type methods because it is simple and economical, and it depicts a physically reasonable link between large-scale moisture convergence and cumulus convection. Tropical cyclone models to date also have frequently employed the Kuo (or moisture convergence) scheme (see Anthes 1982, Table 4.1.). The basic idea behind the Kuo convective parameterization is that the moisture supply by large-scale convergence and evaporation from the surface maintains penetrative cumulus convection. The cumulus heating and moistening imparted to the large-scale environment is assumed to be proportional to the temperature and water vapor mixing ratio difference between the environment and the cloud. Although a one-dimensional cloud model (Anthes 1977a) can be used to estimate the cloud properties, they are usually

taken as those on the moist adiabat constructed using a saturation point at a low level.

The 1965 Kuo scheme tended to yield too much moistening, hence, it underestimated convective heating and convective precipitation rates. Kuo (1974) corrected this deficiency by including the moistening parameter b , the fraction of total moisture convergence that is stored in the air to increase the humidity. He stated that in the areas of low-level convergence in the tropics the parameter b would be much smaller than one, but he did not provide a method for evaluating b . Because a proper determination of the moistening parameter is important for obtaining realistic heating and moistening rates, much effort has been focused on the estimation of b within the framework of the 1974 Kuo-scheme formulation (Anthes 1977a; Molinari 1982; Krishnamurti et al. 1983; Geleyn 1985; Molinari 1985).

In recent years it has been recognized that in tropical clouds the presence of cloud water has a significant impact on cloud buoyancy (Betts 1982, 1986). In this spirit, Betts (1986) proposed a new convective adjustment scheme that is based on observations. Baik et al. (1990a, hereafter Part I) incorporated the convective parameterization scheme in an axisymmetric tropical cyclone model and showed that the scheme is capable

Corresponding author address: Dr. Sethu Raman, North Carolina State University, Dept. of Marine, Earth, and Atmospheric Sciences, Box 8208, Raleigh, NC 27695-8208.

of simulating the developing, rapidly intensifying, and mature stages of a tropical cyclone starting from a weak vortex. With the axisymmetric tropical cyclone model employing a moist convective adjustment scheme (Kurihara 1973), Kurihara (1975) found a correlation of 0.75 between the horizontal moisture convergence in low levels and the area-mean precipitation rate, implying a similarity between the moist convective adjustment scheme and any cumulus parameterization scheme based on the low-level moisture convergence. The objective of this study is to compare the evolution of a tropical cyclone simulated in an axisymmetric model using the Betts convective adjustment scheme with that simulated using the Kuo scheme. Tropical cyclone simulations provide a rigorous test of a cumulus parameterization scheme since the energy to drive a storm comes almost entirely from latent heat release in deep cumulus clouds, and the dynamic and thermodynamic structures of an evolving storm are sensitive to the magnitude and vertical distribution of latent heating.

In section 2 the numerical model is summarized, and in section 3 the basic frameworks of the Betts and Kuo parameterizations are described, including some minor modifications to the Betts parameterization used in Part I. The simulations using the Betts and Kuo parameterizations are compared in section 4.

2. Model description

The tropical cyclone model used in this study is identical to that of Part I, except for a few minor modifications that will be described later. Therefore, only a brief summary of the model and the physical processes are described here. The numerical model includes the conservation equations for momentum, mass, energy, water vapor, and the equation of state. The vertical momentum equation is assumed to be hydrostatic. The system of equations is written with σ coordinate in the vertical and axisymmetric polar coordinates in the horizontal on an f plane. The model atmosphere is divided into 15 layers with nonuniform thickness and has an equally spaced horizontal resolution of 20 km. The horizontal domain size is 1000 km. This relatively small horizontal domain can be used by implementing a spectral radiation boundary condition, which employs a different gravity-wave speed for each vertical mode. The governing equations are solved numerically using a second-order finite-difference method.

The model contains subgrid-scale horizontal and vertical diffusion, air-sea interaction, simple radiation, grid-scale phase change (which is described in more detail later in this section), dry convective adjustment, and parameterized moist convective processes with either the Betts scheme or Kuo scheme. The air-sea interaction processes of momentum, heat, and water vapor are parameterized using the bulk aerodynamic method. The exchange coefficient for heat and water

vapor is assumed to be equal to that of momentum, which is calculated iteratively assuming that the lowest-level wind is in the constant stress layer.

The initial tangential wind structure is given by

$$v = v_m \left[\frac{2(r/r_m)}{1 + (r/r_m)^2} \right] \left[\frac{3(\sigma/\sigma_m)}{2 + (\sigma/\sigma_m)^3} \right], \quad (2.1)$$

where v is the tangential wind speed and r is the radius in polar coordinates. The v_m is the maximum tangential wind, r_m the radius of maximum wind, and σ_m the σ level of maximum wind. The v_m , r_m , and σ_m are specified as 10 m s^{-1} , 210 km, and 0.9, respectively. These values of v_m and r_m are close to the observed values for an intensifying tropical depression (Zehr 1976). There is no radial wind initially. In Part I, a v_m value of 7 m s^{-1} was used. This was increased to 10 m s^{-1} to help reduce the time needed for the storm to intensify.

At the model lateral boundary, the initial temperature profile is taken from the mean tropical clear areas sounding in the western Pacific (Gray et al. 1975) and the initial surface pressure is set to 1008.7 mb. The hydrostatic and gradient-wind equations are used to determine the initial surface-pressure and temperature distributions inside the lateral boundary. The initial moisture field is obtained from the mean tropical cluster environment sounding in the western Pacific (Gray et al. 1975). A Gaussian-type perturbation with the amplitude of 10% in relative humidity and e -folding radius of 200 km is added to the moisture field near the disturbance to reduce the integration time before the storm develops. The sea surface temperature is specified as 28°C and the Coriolis parameter is evaluated at 20°N . For further details of the numerical model see Part I.

As described in Part I, the grid-scale phase-change scheme includes a crude representation of the affect of liquid water. This is accomplished by allowing the excess water vapor that condenses at a grid point to evaporate and cool the next lower model layer. When that layer becomes saturated, the process is repeated until the lowest model layer is reached. This process will be referred to as *evaporation* and should not be confused with evaporation from the sea surface. An alternative to the above representation of liquid water is to assume that all of the condensed water vapor at a level immediately falls out as rain. Because the interaction between the parameterized and grid-scale heating plays a critical role in the development of the tropical cyclone (as shown in Part I), both of the above treatments of grid-scale heating (with and without evaporation) will be considered in the comparison of the Betts and Kuo parameterizations. In a more general model that includes a prediction equation for liquid water, the heating and drying by the grid-scale heating would probably lie between the two limiting cases used here.

In Part I, it was shown that the model could simulate the development of a tropical cyclone without the in-

clusion of parameterized heating, but the intensity of the vortex was considerably weaker than when the parameterized heating was included. In the simulation without parameterized heating in Part I, condensation and evaporation were included in the grid-scale heating. That simulation was repeated without evaporation in the grid-scale heating, and the vortex evolution was qualitatively similar to the case with evaporation. However, the evaporation has a significant effect on the heating and drying profiles produced by the grid-scale scheme. This can be seen in Fig. 1, which shows vertical profiles of the grid-scale heating at $t = 24$ h when the maximum tangential wind is still only approximately 10 m s^{-1} in each simulation. The initial effect of the evaporation (when a grid point first becomes saturated) is to cool the layers below the point that has become saturated. However, the lower layers also become more moist and thus are more likely to reach saturation later in the simulation. In Fig. 1, it can be seen that the lowest model level has become saturated for the evaporation case. Thus, the longer-term effect of including evaporation is to increase the moisture content in the lowest model levels and to lower the level of the maximum grid-scale heating.

3. Cumulus parameterizations

Both the Betts and Kuo cumulus parameterizations are applied to conditionally unstable layers, which are

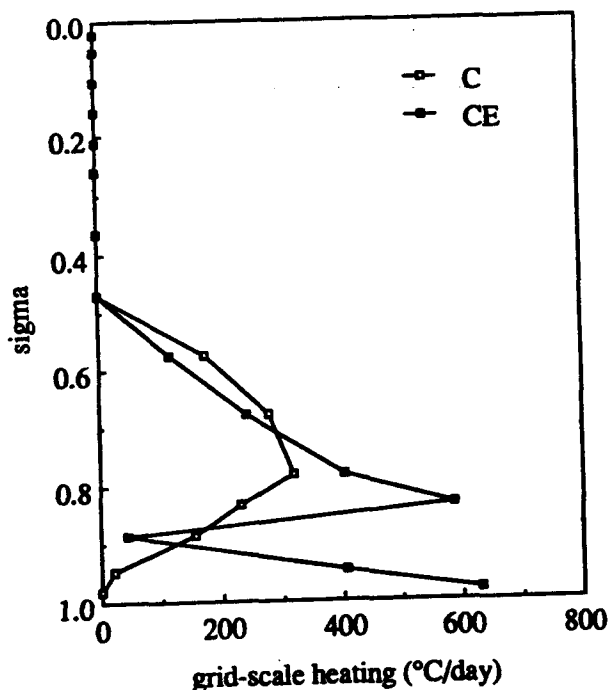


FIG. 1. Vertical profiles of the grid-scale heating at the radius of maximum heating at 24 h for simulations without parameterized convection. The grid-scale heating contains condensation only (C) or condensation and evaporation (CE).

found by the buoyancy check between the environmental (gridpoint) temperature and the temperature that an air parcel would have if lifted from the lowest model level to saturation and then along a moist-adiabatic line. Cloud top is the last level before the lifted parcel first loses buoyancy. Cloud base is fixed as level 14 (about 960 mb for a surface pressure of 1009 mb) because of the relatively low vertical resolution in the lower model atmosphere. At the lowest model level and cloud-base level, no buoyancy test is made. The deep cumulus convection scheme is applied when the calculated cloud top is above level 10 (~ 700 mb).

a. The Betts convective adjustment scheme

The basic idea behind the Betts convective adjustment scheme is that, in the presence of cumulus convection, the local thermodynamic structures are constrained by cumulus convective activity and adjusted toward an observed quasi-equilibrium state. The heating and moistening terms due to cumulus convection can be expressed by

$$F_T = \frac{T_{\text{ref}} - \bar{T}}{\tau}, \quad (3.1)$$

$$F_q = \frac{q_{\text{ref}} - \bar{q}}{\tau}, \quad (3.2)$$

respectively. Here, T is the temperature, q the water vapor mixing ratio, and τ the adjustment (or relaxation) time scale. The subscript ref indicates the reference state and the overbar gridpoint value before cumulus convection occurs. The essence of the Betts scheme lies in the construction of reference thermodynamic profiles.

A crucial observational basis for the deep scheme is that, in the existence of penetrative deep convection, a quasi-equilibrium temperature profile below the freezing level closely parallels a moist virtual adiabat, which includes a parcel buoyancy correction due to cloud water, rather than a moist adiabat. For the deep scheme, first-guess reference profiles are constructed and then are corrected to satisfy the conservation of moist static energy. If the computed convective precipitation rate is found to be negative, the F_T and F_q terms are set to zero at that column.

The slope of the reference potential temperature profile below the freezing level is assumed to be some fraction w times that of a moist adiabat so that

$$\left(\frac{\partial \theta}{\partial p}\right)_{\text{ref}} = w \left(\frac{\partial \theta}{\partial p}\right)_m, \quad (3.3)$$

where θ is the potential temperature and p the pressure. The subscript m indicates that a quantity is evaluated on a moist adiabat and ref indicates the reference profile. The reference moisture profile is determined by specifying the saturation pressure departure at the

lowest model integer level (S_a). The saturation pressure departure is assumed to linearly increase (in magnitude) to a value of $1.25S_a$ at the freezing level and then to linearly decrease to $0.75S_a$ at cloud top.

The application of the Betts scheme requires the specification of the adjustment time scale τ in (3.1) and (3.2), the stability weight w in (3.3), and the saturation pressure departure S_a . In Part I, these were specified as $\tau = 2$ h, $w = 0.95$, and $S_a = -30$ mb. In the current study, the same value of S_a is used as in Part I, but the choices of τ and w have been modified. Betts (personal communication, 1990) has suggested that the choice of τ should depend on the grid resolution and that $\tau = 2$ h is more appropriate for the resolution of global models. For this reason, $\tau = 0.5$ h was used in all of the simulations presented here. The stability weight w has been modified by assuming that the reference temperature profile is exactly equal to that on a moist virtual adiabat. Using the relationship between a moist adiabat and a moist virtual adiabat described by Betts (1982), w is given by

$$w(p, T) = 1 - \frac{c_p R_v T^2 (p - e_s)}{LR_v T (p - e_s) + \epsilon^2 L e_s (L + R_v T)} \quad (3.4)$$

Here c_p is the specific heat for air at constant pressure, R_v the gas constant for water vapor, e_s the saturation water vapor pressure, L the latent heat of evaporation, and $\epsilon = 0.622$. For a wide range of pressure (500 mb, 1000 mb) and temperature (0°C , 30°C), w computed using (3.4) lies between 0.90 and 0.93. Baik et al. (1990b) showed that the evolution of the model storm is not very sensitive to τ and w , therefore, the above changes in the Betts parameterization are relatively minor relative to the version used in Part I.

The Betts parameterization described in Part I also allows for shallow nonprecipitating convection. Because the Kuo parameterization does not account for shallow convection, the shallow convective parameterization was eliminated from the Betts scheme in all of the results that will be presented. To determine the effect of neglecting the shallow convection, the control simulation described in Part I was repeated without shallow convection. These results showed that, in the simulation without shallow convection, the storm intensified about 12 h later than the simulation with shallow convection, but otherwise, the results were very similar. Thus, the shallow convective parameterization plays only a minor role in the vortex evolution. The above result appears to contradict the results of Emanuel (1989), which showed that shallow convection is fundamental in a simple three-layer hurricane model. However, some of the effects of the parameterized shallow convection in the three-layer model may be represented by resolvable scales in the current model. Also, in Emanuel (1989), both shallow and deep convection can exist at a grid point, while in the Betts

formulation only one type of convection can exist at each grid point. As shown in Part I, the inner 300 km of the model domain was dominated by parameterized deep convection during the first 4 days of the simulation, while shallow convection prevailed outside of this region. Thus, it might be expected that the elimination of the shallow convection would have only a minor effect on the vortex evolution.

b. The Kuo convective parameterization schemes

To incorporate collective effects of deep cumulus convection on the large-scale temperature and moisture fields, Kuo (1965) assumed that cumulus convection takes place in deep layers of conditionally unstable atmosphere and cumulus clouds generated dissolve instantaneously through lateral mixing with the environment, thereby imparting the heat and moisture carried by cloud air to the surroundings. The heating and moistening of the environment that result from the mixing of cumulus clouds with the surrounding air are given by

$$Q_T = C_{pr}(T_c - T_e), \quad (3.5)$$

$$Q_q = C_{pr}(q_c - q_e), \quad (3.6)$$

respectively. Here subscripts c and e denote quantities inside cloud and of the environment. The C_{pr} is the total cloud production rate per unit time. Note that q_c is the saturation mixing ratio at the temperature T_c . The total cloud production rate is assumed to be proportional to the total moisture accession rate per unit horizontal area (denoted by M_i), which is the sum of horizontal moisture convergence and surface evaporation, and inversely proportional to the total amount of water vapor necessary for producing cloud over a unit area (denoted by W). These are expressed by

$$C_{pr} = \frac{M_i}{W}, \quad (3.7)$$

$$M_i = -\frac{1}{gr} \frac{\partial}{\partial r} \int_{p_n}^{p_a} (ruq) dp + E_s, \quad (3.8)$$

$$W = \frac{1}{gL} \int_{p_n}^{p_a} [c_p(T_c - T_e) + L(q_c - q_e)] dp, \quad (3.9)$$

where g represents the acceleration of gravity, p_a the pressure at the lowest model level, p_i the pressure at cloud top, u the radial wind speed, and E_s the surface evaporation.

In 1974 Kuo proposed the partition of the total moisture accession rate in a way that a fraction $(1 - b)M_i$ is condensed and either precipitated out or carried away, while the remaining fraction bM_i increases the relative humidity of the air column. The convective heating and moistening terms in the 1974 version are represented by

$$\left(\frac{\partial T}{\partial t}\right)_{\text{conv}} = \frac{gL(1-b)M_i(T_c - T_e)}{c_p(p_a - p_i) \langle T_c - T_e \rangle}, \quad (3.10)$$

$$\left(\frac{\partial q}{\partial t}\right)_{\text{conv}} = \frac{gbM_i(q_c - q_e)}{(p_a - p_i) \langle q_c - q_e \rangle}, \quad (3.11)$$

respectively, where quantities in the angle brackets denote the values averaged from $p = p_i$ to $p = p_a$. As mentioned in the Introduction, a proper evaluation of moistening parameter b is important. In this study a method proposed by Anthes (1977a) is employed to estimate b . The basic idea of the method is that environmental moistening associated with cumulus convection is strong when the atmosphere is dry and the moistening is weak when wet. The parameter b is assumed to be a function of mean relative humidity in the air column:

$$b = \begin{cases} \left[\frac{1 - \langle \text{RH}_e \rangle}{1 - \text{RH}_c} \right]^n & \text{if } \langle \text{RH}_e \rangle \geq \text{RH}_c \\ 1 & \text{if } \langle \text{RH}_e \rangle < \text{RH}_c, \end{cases} \quad (3.12)$$

where $\langle \text{RH}_e \rangle$ is the mean environmental relative humidity in the air column, RH_c a critical relative humidity, and n a positive exponent. When $\langle \text{RH}_e \rangle$ is less than RH_c , only environmental moistening takes place. Anthes (1977a) also proposed a slightly different formulation of vertical partitioning of convective moistening. This is given by

$$\left(\frac{\partial q}{\partial t}\right)_{\text{conv}} = \frac{gbM_i(100\% - \text{RH})q_s(T_e)}{(p_a - p_i) \langle (100\% - \text{RH})q_s(T_e) \rangle}, \quad (3.13)$$

where RH is the relative humidity. Cumulus convective processes are activated when the computed cloud-top height is above level 10 and the vertically integrated moisture convergence rate exceeds a critical value of $10^{-5} \text{ kg m}^{-2} \text{ s}^{-1}$ (Anthes 1977b). The temperature and mixing ratio inside cloud, T_c and q_c , are considered as those on a moist adiabat passing through the lifting condensation level of an air parcel at the lowest model level.

The three basic versions of the Kuo scheme described above are denoted by:

- K65 ; using (3.5)–(3.9),
- KA1 ; using (3.8), (3.10)–(3.12),
- KA2 ; using (3.8), (3.10), (3.13), and (3.12).

The inclusion of the moistening parameter b is generally considered an improvement to the Kuo scheme, therefore, K65 will not be discussed further. For the KA1 and KA2, it is necessary to choose n and RH_c in (3.12). To test a cumulus parameterization scheme utilizing a one-dimensional, steady-state cloud model (Anthes 1977a), Anthes (1977b) chose values of $n = 1$ and $\text{RH}_c = 0.5$ for the axisymmetric hurricane model. These values were adopted by Hobgood and Rayner (1989) in testing Kuo-type convective parameteriza-

tions in a tropical cyclone model. When the vertically averaged environmental relative humidity $\langle \text{RH}_e \rangle$ varies between 95% and 80%, b ranges from 0.1 to 0.4. However, a number of studies have indicated that the b value computed with $n = 1$ and $\text{RH}_c = 0.5$ is large, and consequently convective heating, drying, and rainfall rates are underestimated. In a study on semi-prognostic tests of Kuo-type schemes in an extratropical convective system, Kuo and Anthes (1984) found that the simulated rainfall rate agrees best with observations when n is between 2 and 3, and RH_c is between 0.25 and 0.50. Using a semiprognostic approach, Someshwar Das et al. (1988) found that, during the Asian summer monsoon periods, the best results are obtained for values of n between 3 and 5, and RH_c between 0 and 0.25. In the ECMWF numerical forecasting model, values of $n = 3$ and $\text{RH}_c = 0$ are used (Tiedtke 1986). As a compromise between these values, n and RH_c are specified as 3 and 0.5, respectively.

Several sensitivity tests were performed where n was varied using the KA1 version of the parameterization, and the KA1 and KA2 versions of the parameterizations were compared. These results showed that in all cases the model produced an intense tropical cyclone that reaches the mature stage by about 144 h. There were some minor differences in the simulations, namely, the minimum surface pressure at the mature stage varied by a few millibars and the time when the storm began to intensify rapidly varied by up to 12 h. However, the storm evolution was qualitatively similar using the KA1 or KA2 parameterizations. In the comparison with the Betts parameterization, the KA1 version of the parameterization with $n = 3$ and $\text{RH}_c = 0.5$ is considered.

4. Comparisons of the Betts scheme with the Kuo scheme

In this section, similarities and differences in the model storm evolution fields simulated with the Betts convective adjustment scheme and the Kuo convective parameterization are discussed. The simulations with the Betts and Kuo schemes were run with and without evaporation included in the grid-scale heating. In this section, the following notation will be used to describe the model simulations.

- K.C ; KA1 version of Kuo scheme, grid-scale condensation,
- K.CE ; KA1 version of Kuo scheme, grid-scale condensation, and evaporation,
- B.C ; Betts scheme, grid-scale condensation,
- B.CE ; Betts scheme, grid-scale condensation, and evaporation.

a. Storm evolution

Figure 2 shows the evolution of the maximum lowest-level tangential winds for the simulations with the

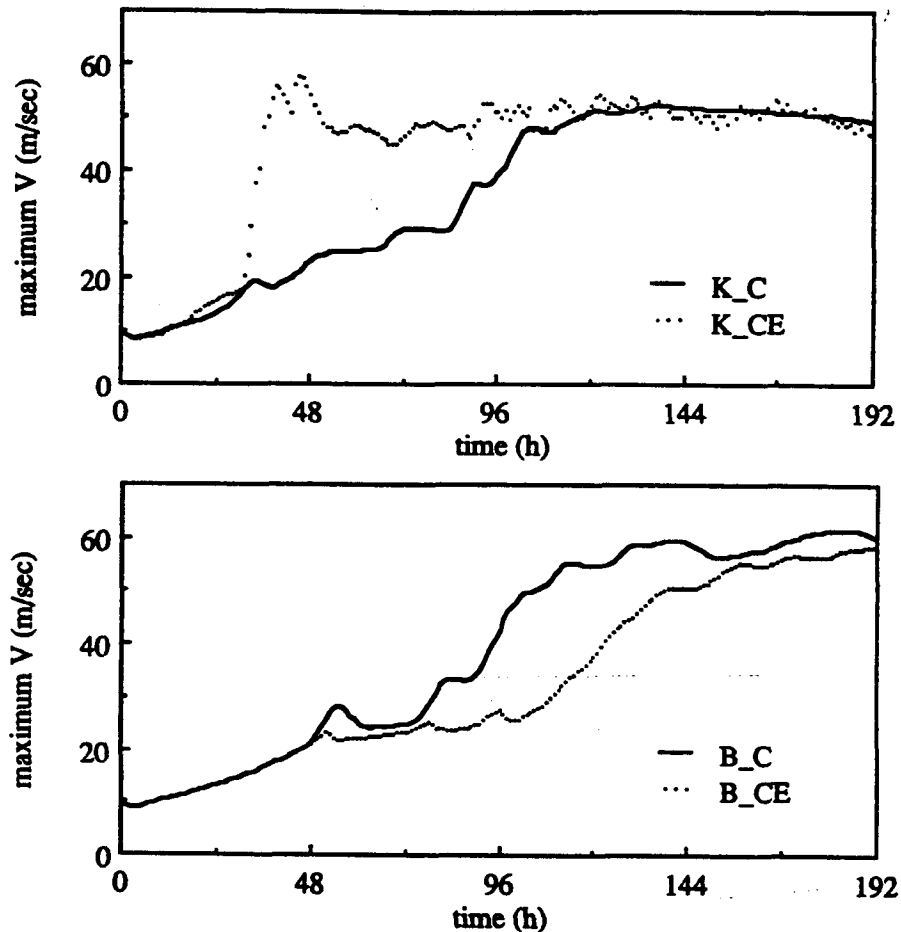


FIG. 2. The time evolution of the maximum lowest-level tangential wind in simulations with the Kuo (K.C and K.CE) and Betts (B.C and B.CE) convective parameterizations. The C indicates the grid-scale heating included condensation only, and CE indicates that condensation and evaporation were included.

Kuo and Betts schemes. The storm evolution for the simulations without evaporation (K.C and B.C) are similar. In both cases, the storm intensifies slowly for about three days, rapidly intensifies for about two days, and reaches a quasi-steady state thereafter. The final intensity of the storm is slightly higher for the simulation with the Betts scheme. This is also the case when evaporation is included.

When evaporation is included, the storm evolutions with the Kuo and Betts schemes are dramatically different. The inclusion of evaporation with the Kuo scheme causes much earlier and more rapid storm development. With the Betts scheme, the inclusion of evaporation has the opposite effect, resulting in later and less rapid intensification. Despite the changes in the storm evolution, the storms reach approximately the same intensity with and without grid-scale evaporation.

Figure 3 shows the time evolution of the radius of maximum tangential wind for each of the four simu-

lations shown in Fig. 2. Figure 3 shows that the storm expands initially (more for the Betts simulations) as the convection becomes organized and then contracts during the rapid intensification stage. There is also a tendency for the storm to expand during the mature stage, especially for the K.CE simulation that reaches the mature stage after about 72 h.

The minimum value of the radius of maximum winds in each of the simulations is approximately 70 km, which occurs after the periods of rapid intensification. This is relatively large considering that the horizontal resolution of the model is 20 km. This is probably due to the large initial vortex as defined by (2.1) as well as the particular choices of the horizontal and vertical diffusion parameters described in Part I.

b. Parameterized and grid-scale precipitation

Table 1 lists the fraction of the inner 500-km domain-averaged and 24-h time-averaged parameterized

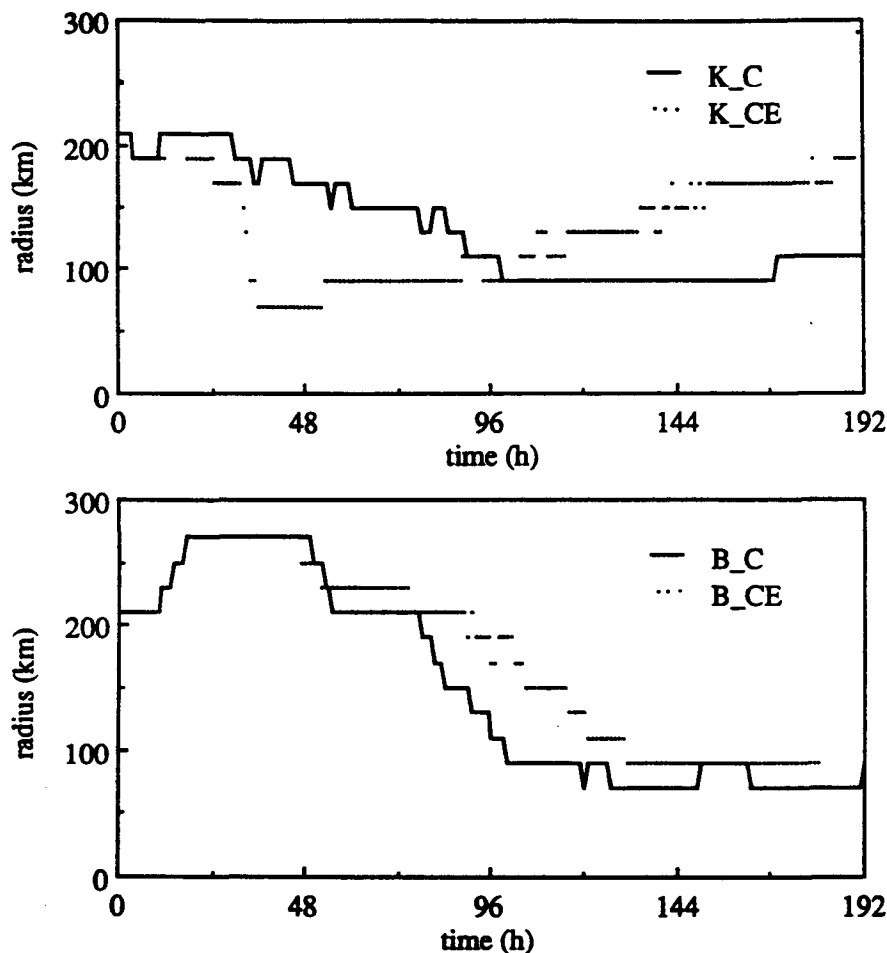


FIG. 3. The time evolution of the radius of maximum lowest-level tangential wind for the K.C, K.CE, B.C, and B.CE simulations.

to the total precipitation rates. There are some general similarities in all four simulations because most of the initial precipitation comes from the parameterization, and most or all of the precipitation at the mature stage

TABLE 1. The fraction of the parameterized to the total precipitation rates for the Kuo (K.C and K.CE) and Betts (B.C and B.CE) simulations. For this calculation, the parameterized and total precipitation rates are averaged over the inner 500-km domain and the time interval of 24 h. The underline values indicate the time periods when the vortex is intensifying rapidly (see Fig. 2).

Time interval (h)	K.C	K.CE	B.C	B.CE
0-24	0.79	0.87	1.00	1.00
24-48	0.44	<u>0.30</u>	0.90	0.89
48-72	0.24	0.08	0.44	0.82
72-96	<u>0.15</u>	0.15	<u>0.40</u>	0.78
96-120	<u>0.06</u>	0.18	<u>0.16</u>	<u>0.74</u>
120-144	0.06	0.15	0.07	<u>0.45</u>
144-168	0.05	0.10	0.01	0.30
168-192	0.06	0.09	0.00	0.07

results from the grid-scale scheme. Comparing Table 1 with Fig. 2 shows that during the rapid intensification periods a substantial fraction of the precipitation is from the grid-scale scheme. For the Betts simulations, a larger fraction of the precipitation is due to the parameterization initially and to the periods of rapid intensification. At the mature stage, however, a smaller fraction of the precipitation is due to the parameterization for the Betts scheme relative to the Kuo scheme. For both the Kuo and Betts schemes, the inclusion of grid-scale evaporation increases the fraction of the precipitation due to the parameterizations during the rapid intensification and mature stages. This should be expected since there is more water vapor available to the parameterizations when evaporation of grid-scale liquid water is included.

c. Vertical heating profiles

The above results showed that at the mature stage the storms with both the Betts and Kuo schemes are

maintained by latent heat release from the grid-scale schemes. However, in Part I it was shown that the vortex was weaker and intensified much more slowly when the parameterized convection was not included. This indicates that the parameterizations are playing an important role in the storm development even though they become negligible at the mature stage. To further understand the interaction between the grid-scale and parameterized heating, it is necessary to consider the time evolution of the vertical heating profiles as shown in Fig. 4. The profiles in Fig. 4 are at the radius of maximum total heating, which is close to the radius of maximum wind shown in Fig. 3.

At 24 h, the difference between the K.C and K.CE simulations is already apparent. There is low-level grid-scale heating in the K.CE simulation, indicating that the evaporation has moistened the low levels. This feeds back into the Kuo parameterization, which results in heating through a deep layer. Once the parameterized heating becomes active, vertical motion results that enables the grid-scale heating to occur through a deep layer. When this happens, the storm intensifies rapidly as was seen in Fig. 2 for the K.CE simulation. In the K.C simulation, the parameterized heating is active at other radii at 24 h, but the magnitude is much smaller than in the K.CE case. Thus, the feedback process with the grid-scale heating takes much longer to occur. By 84 h, the level of maximum grid-scale heating in the K.C case has increased, and the storm is about to intensify rapidly. In the K.CE simulation, the storm is well into the mature stage by 84 h, where the grid-scale heating dominates.

The above result shows that the Kuo parameterization is extremely sensitive to the choice of the grid-scale scheme. This is because the magnitude of the parameterized heating is proportional to the moisture convergence. In a hurricane simulation where the low-level convergence is quite large (radial velocities can exceed 10 m s^{-1} in the boundary layer as shown in Part I), processes that affect the low-level moisture distribution can have dramatic effects as shown in Figs. 2 and 4. The extreme sensitivity to evaporation of liquid water also suggests that a predictive equation for liquid water should be included. In particular, the grid-scale heating profiles are not realistic for the case when evaporation is included. The large grid-scale heating in the lowest model level at 84 h in the K.CE simulation indicates that the boundary-layer moisture budget is extremely inaccurate.

In Fig. 4, it can be seen that the heating profiles in the B.C and B.CE simulations are identical at 24 h since the grid-scale heating has not yet been activated. This indicates that the Betts parameterization is more effective at removing water vapor and preventing grid-scale saturation. In the Betts simulations, the domain-averaged precipitation from the parameterization was approximately three times as large as that from the Kuo parameterizations during the first 24 h of the sim-

ulations. This is consistent with the results of Mesinger et al. (1990), which showed that the Betts scheme produced higher precipitation amounts relative to the Kuo scheme in a three-dimensional mesoscale forecast model.

By 84 h in the B.C simulation, the grid-scale heating has become active through a deep layer, and rapid intensification is about to take place. In the B.CE simulation, the grid-scale heating is still relatively small (it is zero at the radius of maximum heating) at 84 h. The grid-scale heating does become active by about 108 h in the B.CE simulation, and rapid intensification takes place shortly thereafter. Thus, the inclusion of evaporation has delayed the intensification in the Betts simulations, which is opposite to the effect of evaporation in the Kuo simulations. This is because the Betts scheme does not directly respond to changes in the moisture convergence, therefore, the feedback process described for the Kuo scheme does not occur. For the Betts scheme the evaporation makes more water vapor available to the parameterization and reduces the grid-scale heating. Thus, the rapid intensification is delayed when evaporation is included.

The above results show that the interaction between the parameterized and grid-scale heating is critical for understanding the intensification of the model storms. Figure 5 shows the vertical heating profiles produced by parameterizations at the radius of maximum heating at the time prior to the rapid intensification. The times were chosen when the parameterized heating was near its maximum. In all cases, the magnitude of the parameterized heating decreases during the mature stage as discussed previously. Figure 5 shows that the shapes of the profiles are relatively similar, although the Kuo scheme has a tendency to produce heating through a slightly deeper layer than the Betts scheme. On the other hand, the Betts scheme appears to produce more heating in the lower layers.

Numerous studies (e.g., Hack and Schubert 1986) have shown that the response of a vortex is sensitive to the vertical structure of the diabatic heating. For heating profiles that extend through a deep layer, a vortex will intensify more rapidly when the level of maximum heating moves toward lower levels. There is some tendency in Fig. 5 for the level of maximum heating to be lower in the B.C simulation relative to the K.C simulation, which would favor more rapid intensification with the Betts parameterization. This might be important in a model with coarser horizontal resolution where the grid-scale heating is less important. In the current simulations, much larger variations in the vertical structure and magnitude of the total heating result when the grid-scale heating becomes active.

d. Storm structure at the mature stage

Despite the large differences in the timing of the intensification of the storms as shown in Fig. 2, the storm

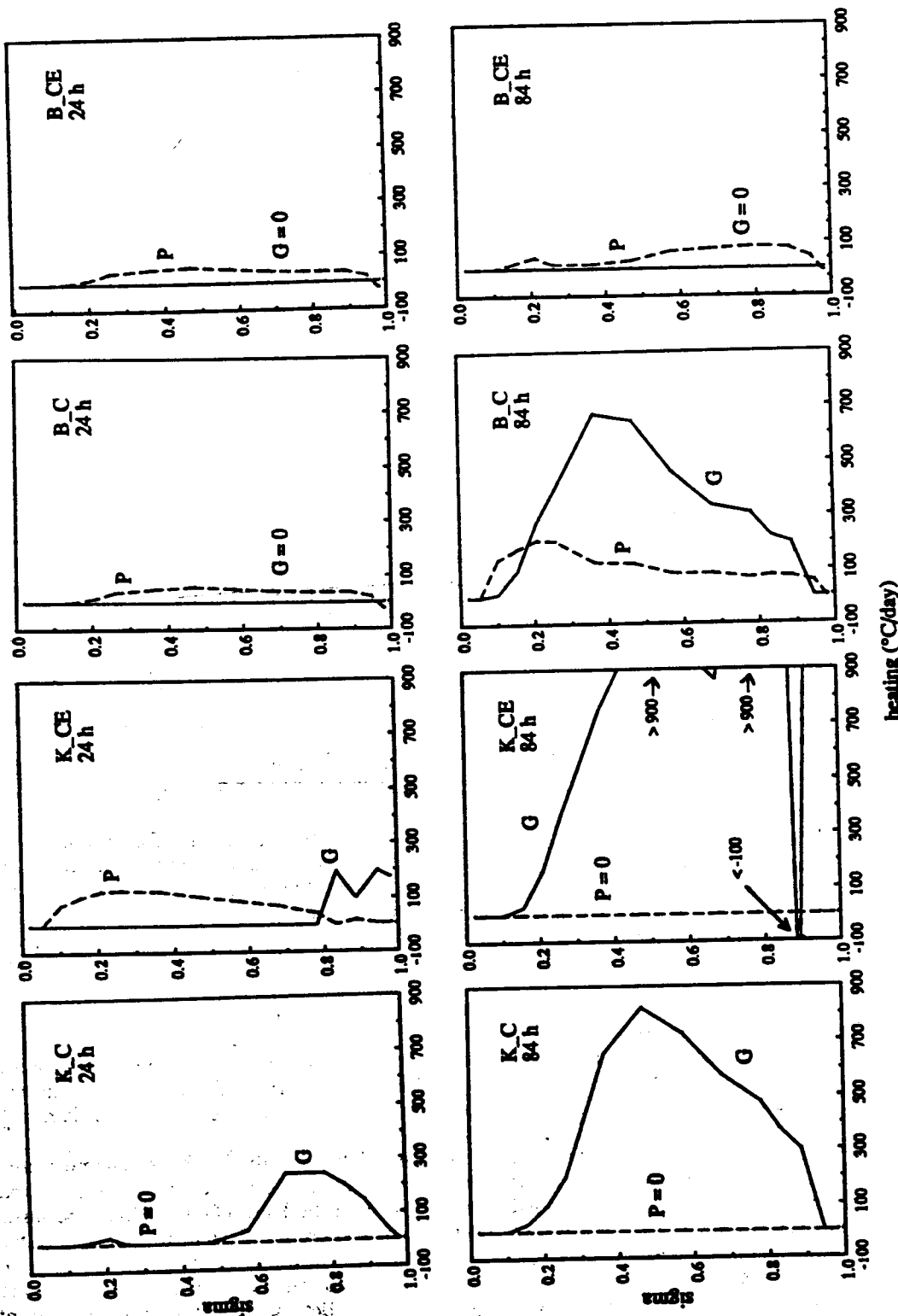


FIG. 4. Vertical profiles of the parameterized (P) and grid-scale (G) heating at the radius of the maximum total heating at $t = 24$ and 84 h for the K_C, K_CE, B_C, and B_CE simulations.

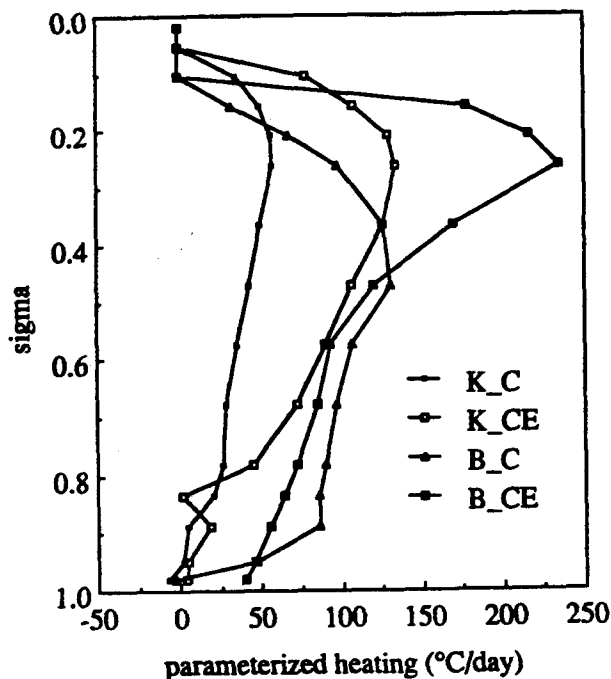


FIG. 5. Vertical profiles of the parameterized heating at the radius of maximum parameterized heating. The times were chosen when the parameterized heating was a maximum, but prior to the rapid intensification stage (at 12, 24, 72, and 96 h for the K.C, K.CE, B.C, and B.CE, respectively).

structures at the mature stage were quite similar to those described in Part I. This is not surprising since the storms are primarily maintained by the grid-scale heating by the end of the simulations, as was shown in Table 1. There were, of course, some differences in structure related to variations in the final intensity and radius of maximum wind. The largest difference in structure occurred for the K.CE simulation that had a significantly larger vortex at 192 h. This was because the storm in the K.CE simulation reached the mature stage much sooner than in the other simulations.

5. Concluding remarks

An axisymmetric tropical cyclone model was used to investigate the essential differences between the Betts convective adjustment scheme and the Kuo scheme through numerical simulations. The simulations also included two versions of the grid-scale latent heat release. In one version, evaporation was neglected so that all of the condensed water vapor immediately fell out as rain. In a second version, evaporation was included by allowing the condensed water vapor to evaporate into the next lower model level. It was shown that the inclusion of evaporation in the grid-scale heating eventually saturates the lowest model levels and results in a grid-scale heating profile with a lower level of maximum heating than the simulation without evaporation.

In section 3 the basic frameworks of the Betts and Kuo schemes for parameterizing deep cumulus convective processes were presented. The principal idea of the Betts scheme is that in the presence of cumulus convection the local thermodynamic structures are constrained by cumulus convection and adjusted toward an observed quasi-equilibrium thermodynamic state. A number of minor modifications to the Betts scheme used in Part I were described in this section. The basic idea of the Kuo scheme is that the moisture supply by large-scale convergence and evaporation from the surface maintains cumulus convection. Three versions of the Kuo scheme were described: the 1965 Kuo scheme; the 1974 Kuo scheme; and the modified Kuo scheme by Anthes (1977a). The moistening parameter b was calculated using a method proposed by Anthes (1977a) in which the parameter b is a function of mean relative humidity in the air column. The 1974 Kuo scheme was used to compare with the Betts scheme since the vortex evolution was similar in model simulations with 1974 Kuo and modified Kuo schemes.

In section 4, the results from simulations with the Betts convective adjustment scheme and the 1974 Kuo parameterization were presented. It was shown that the storm with the Betts scheme has a slightly more intense mature stage than the storm with the Kuo scheme. For both the Betts and Kuo schemes, the storm was primarily maintained by the grid-scale heating at the mature stage, while parameterized convection dominated initially. It was also shown that the storms did not intensify rapidly until the grid-scale heating became a significant fraction of the total heating. Thus, the interaction between the parameterized and grid-scale heating was crucial for the development of the storm.

It was shown that inclusion of evaporation in the grid-scale scheme had a dramatic effect on the storm evolution when the Kuo scheme was used. When evaporation was included, the moisture in the lowest levels increased. This then increased the boundary-layer moisture convergence, and the Kuo scheme produced a stronger deep convective heating profile relative to the Kuo-scheme simulation without evaporation. This then allowed the grid-scale heating to occur through a deep layer, and the storm developed rapidly. With the Kuo scheme, the storm with the evaporation reached the mature stage almost three days sooner than without evaporation.

The simulations with the Betts scheme were less sensitive to the inclusion of the grid-scale evaporation than those with the Kuo scheme. This is because the Betts scheme was more effective at removing water vapor and delayed the onset of the grid-scale heating relative to the Kuo scheme. When evaporation was included with the Betts scheme, the onset of rapid intensification was delayed. This is opposite to the effect of grid-scale evaporation with the Kuo scheme.

The above sensitivity to grid-scale evaporation sug-

gests that a prognostic equation for liquid water should probably be included so that evaporation effects can be treated more accurately. The above sensitivity also indicates that forecasting tropical cyclone intensity changes with numerical models will be a difficult task. For example, the time of rapid intensification began at about 30 h for the Kuo scheme with evaporation, but it began at about 100 h for the Betts scheme with evaporation. A similar reservation concerning uncertainties in tropical cyclone simulations was noted by Rosenthal (1979).

In this study, we compared the Betts convective adjustment scheme with the Kuo convective parameterization scheme only in idealized axisymmetric tropical cyclone model simulations. Recently, Junker and Hoke (1990) investigated the performance of the Betts and 1965 Kuo schemes in predicting heavy rainfall over the southern United States in the cool season using the NMC (National Meteorological Center) Nested Grid Model. The Betts scheme gave favorable precipitation scores, but it tended to produce overdeepening of some low pressure systems. An interesting area of research will be to investigate behaviors of the Betts and Kuo schemes in forecasting heavy rainfall during summer. This research, employing a limited-area nested-grid model to predict Indian southwest monsoon rainfall, is in progress.

Acknowledgments. The authors would like to thank the anonymous reviewers for their critical comments on this study. This research was supported by the National Science Foundation under Grant ATM-8817763. The computer time used in this study was provided by the National Center for Atmospheric Research, which is sponsored by the National Science Foundation.

REFERENCES

- Anthes, R. A., 1977a: A cumulus parameterization scheme utilizing a one-dimensional cloud model. *Mon. Wea. Rev.*, **105**, 270-286.
- , 1977b: Hurricane model experiments with a new cumulus parameterization scheme. *Mon. Wea. Rev.*, **105**, 287-300.
- , 1982: *Tropical Cyclones: Their Evolution, Structure and Effects*. Meteor. Monogr., No. 41, Amer. Meteor. Soc., 208 pp.
- Baik, J.-J., M. DeMaria and S. Raman, 1990a: Tropical cyclone simulations with the Betts convective adjustment scheme. Part I: Model description and control simulation. *Mon. Wea. Rev.*, **118**, 513-528.
- , —, and —, 1990b: Tropical cyclone simulations with the Betts convective adjustment scheme. Part II: Sensitivity experiments. *Mon. Wea. Rev.*, **118**, 529-541.
- Betts, A. K., 1982: Saturation point analysis of moist convective overturning. *J. Atmos. Sci.*, **39**, 1484-1505.
- , 1986: A new convective adjustment scheme. Part I: Observational and theoretical basis. *Quart. J. Roy. Meteor. Soc.*, **112**, 677-691.
- Emanuel, K. A., 1989: The finite amplitude nature of tropical cyclogenesis. *J. Atmos. Sci.*, **46**, 3431-3456.
- Geleyn, J.-F., 1985: On a simple, parameter-free partition between moistening and precipitation in the Kuo scheme. *Mon. Wea. Rev.*, **113**, 405-407.
- Gray, W. M., E. Ruprecht and R. Phelps, 1975: Relative humidity in tropical weather systems. *Mon. Wea. Rev.*, **103**, 685-690.
- Hack, J. J., and W. H. Schubert, 1986: Nonlinear response of atmospheric vortices to heating by organized cumulus convection. *J. Atmos. Sci.*, **43**, 1559-1573.
- Hobgood, J. S., and J. N. Rayner, 1989: A test of convective parameterizations in a tropical cyclone model. *Mon. Wea. Rev.*, **117**, 1221-1226.
- Junker, N. W., and J. E. Hoke, 1990: An examination of nested grid model precipitation forecasts in the presence of moderate-to-strong low-level southerly inflow. *Wea. Forecasting*, **5**, 333-344.
- Krishnamurti, T. N., S. Low-Nam and R. Paach, 1983: Cumulus parameterization and rainfall rates II. *Mon. Wea. Rev.*, **111**, 815-828.
- Kuo, H. L., 1965: On formation and intensification of tropical cyclones through latent heat release by cumulus convection. *J. Atmos. Sci.*, **22**, 40-63.
- , 1974: Further studies of the parameterization of the influence of cumulus convection on large-scale flow. *J. Atmos. Sci.*, **31**, 1232-1240.
- Kuo, Y.-H., and R. A. Anthes, 1984: Semiprognostic tests of Kuo-type cumulus parameterization schemes in an extratropical convective system. *Mon. Wea. Rev.*, **112**, 1498-1509.
- Kurihara, Y., 1973: A scheme of moist convective adjustment. *Mon. Wea. Rev.*, **101**, 547-553.
- , 1975: Budget analysis of a tropical cyclone simulated in an axisymmetric numerical model. *J. Atmos. Sci.*, **32**, 25-59.
- Mesinger, F., T. L. Black, D. W. Plummer and J. H. Ward, 1990: Eta model precipitation forecasts for a period including Tropical Storm Allison. *Wea. Forecasting*, **5**, 483-493.
- Molinari, J., 1982: A method for calculating the effects of deep cumulus convection in numerical models. *Mon. Wea. Rev.*, **110**, 1527-1534.
- , 1985: A general form of Kuo's cumulus parameterization. *Mon. Wea. Rev.*, **113**, 1411-1416.
- Rosenthal, S. L., 1979: The sensitivity of simulated hurricane development to cumulus parameterization details. *Mon. Wea. Rev.*, **107**, 193-197.
- Someshwar Das, U. C. Mohanty and O. P. Sharma, 1988: Study of Kuo-type cumulus parameterizations during different epochs of the Asian summer monsoon. *Mon. Wea. Rev.*, **116**, 715-729.
- Tiedtke, M., 1986: The parameterization of moist processes. Part 2: Parameterization of cumulus convection. ECMWF Lecture Note No. 3.3, 53 pp.
- Zehr, R. M., 1976: Tropical disturbance intensification. Dept. of Atmospheric Science Pap. No. 259, Colorado State University, Ft. Collins, CO, 91 pp.