

# MOMENTUM FLUX AND WAVE SPECTRA MEASUREMENTS FROM AN AIR-SEA INTERACTION BUOY

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**Abstract.** Momentum flux measurements by eddy correlation method and wave height measurements with a capacitance-type wave staff were carried out from a stable air-sea interaction buoy anchored 5 km off Tiana Beach, Long Island, New York State. A characteristic height of sea surface  $h_s$  was estimated from wave height spectra. A roughly linear variation of surface shear stress with  $h_s$  was found for an aerodynamically rough sea surface.

## 1. Introduction

Surface shear stress over the ocean induced by the atmosphere is basically different from that over land, due to the mobility of the underlying surface. Although the flux of momentum is generally from the atmosphere to the ocean, there can be occasions when the momentum transfer takes place in the opposite direction, usually due to the development of swells propagated by a distant storm or by local low wind conditions after the passage of a storm. Restricting ourselves to the common conditions of exchanges of momentum from the atmosphere to the ocean, the variation of fluxes under such conditions over the ocean will be examined in this paper. Measurements of the surface drag coefficient,  $C_D$ , in a previous study indicated that  $C_D$  is largely influenced by the surface parameters of the sea, and is essentially independent of mean wind speed (SethuRaman and Raynor, 1975). Other investigators (Kraus, 1967; Weiler and Burling, 1967) also found  $C_D$  to be nonvariant with mean wind speed, for sea surfaces that were aerodynamically smooth, moderately rough or fully rough. The degree of roughness was characterized by a roughness Reynolds number  $u_* z_0 / \nu$ , where  $u_*$  is the friction velocity,  $z_0$  the roughness length and  $\nu$  the kinematic viscosity. Roughness length in this parameterization was obtained from mean wind profiles. An independent evaluation of the characteristic height of roughness  $h_s$  (which may be related to  $z_0$ ) was thought to be the next step in determining the variation of aerodynamic roughness and hence  $C_D$  over water. This required an estimate of  $h_s$ . Due to the mobility of the water surface, the drag seems to be caused by waves with phase velocities less than the near-surface wind speed. Because this usually involves high frequency waves, a measure of their amplitude would represent the characteristic height of the sea surface. Kondo *et al.* (1973) estimated a characteristic height from the variance spectra of wave heights, but did not allow for any weighting factor between different frequencies that would account for varying contributions. The method adopted here utilizes an exponential relationship that depends on friction velocity and wave-length. This relationship avoids any arbitrary frequency cut-off.

Measurements of momentum flux by eddy correlation technique and of high-frequency wave heights were made from a stable air-sea interaction buoy (SethuRaman *et al.*, 1978) anchored about 5 km off the south shore of Long Island from October 5-8, 1975. The measurement period each day lasted from 2 to 3 h and consisted of different surface wave conditions or wave ages. This provided observations with a wide range of  $C_D$  and  $h_s$ . A meteorological tower at the beach was used to obtain continuous wind direction measurements. Locations of the buoy and the beach tower are shown in Figure 1.

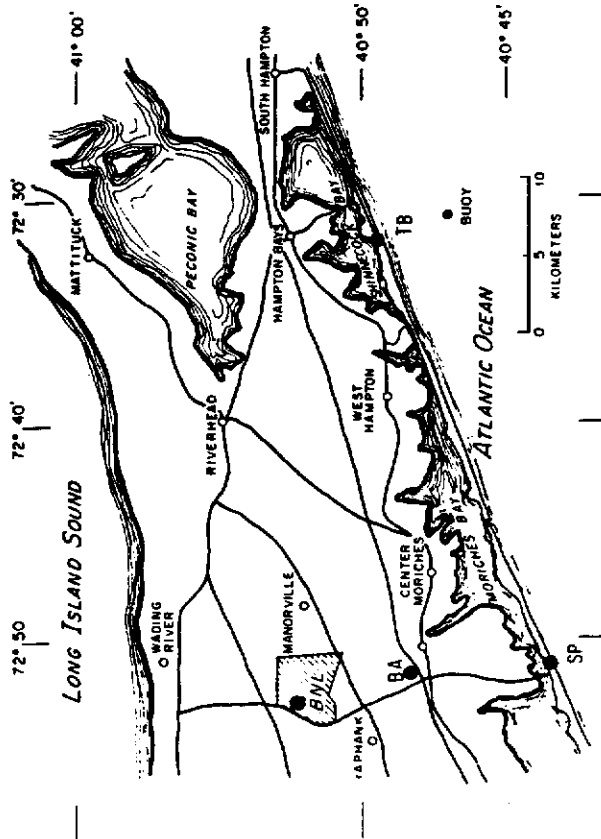


Fig. 1. Site of the over-water experiments. Buoy is about 5 km off shore from Tiana Beach (indicated as TB). A 24-m tower is located at TB.

## 2. Methods

A general view of the instrumented buoy above water is shown in Figure 2. The buoy has over-water and under-water towers and is designed for simultaneous meteorological and oceanographic measurements. The buoy motion was caused only by the dominant swell waves, with maximum values being reached at the frequency of these waves. Due to the substantial weight ( $\approx 9000$  kg), provision of a large damper plate and the cylindrical shape, motion of the buoy in the vertical direction was significantly reduced even at these frequencies and there was hardly any response for waves of higher frequencies. Typical vertical velocities of the buoy for the range of mean wind speeds for the experiments reported here varied from 1 to  $3 \text{ cm s}^{-1}$ . The maximum oscillating tilt of the buoy was around  $\pm 5$  deg for wind speeds in excess of  $15 \text{ m s}^{-1}$ . The mean tilt was near zero. In order to offset even

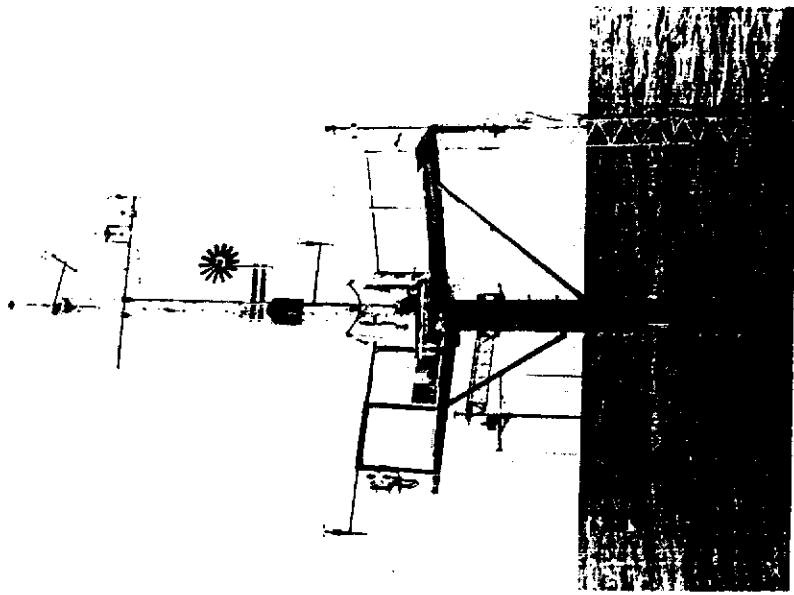


Fig. 2. General view of the instrumented air-sea interaction buoy above water.

these small oscillating tilts, a two-dimensional friction-free gimbal mount mechanical device designed and fabricated to isolate the instrument from its motion, was used.

Momentum flux in the atmospheric surface layer is generally measured by the following methods:

- (1) by eddy correlation technique with longitudinal and vertical velocity fluctuations,
- (2) by mean wind profiles and,
- (3) by the measurement of wind spectra in the inertial subrange.

The mean wind profile method has the disadvantage of requiring an assumed profile relationship, usually logarithmic close to the earth's surface and a value for Karman's constant,  $k$ , generally taken to be 0.4 although a few recent studies (Businger *et al.*, 1971) suggest  $k$  to be around 0.35. The inertial subrange method the problem of assuming values for two constants - Kolmogorov and von Karman would also be affected for atmospheric flow over water by the frequencies in which

wave-induced turbulence is dominant. Hence, of the three methods outlined above, the direct method of obtaining fluxes by the eddy correlation technique seems to be the best, provided the errors due to tilt in the vertical velocity fluctuations are minimized. This was achieved by the use of a two-dimensional gimbal mount, a general view of which is shown in Figure 3. A lead weight was added to the bivane term and the bivane carefully adjusted to have approximately the same area above and below the gimbal plane.

Longitudinal velocity fluctuations,  $u$ , were measured with a single-sensor temperature-compensated vertical hot wire. The metal-clad hot-wire sensor had a frequency response of 5 Hz; the calibrations were found to be repeatable after prolonged use in the atmosphere. Effect of maximum tilt of about 5 deg on the response of the hot wire was found to be negligible. Vertical velocity fluctuations,  $w$ ,

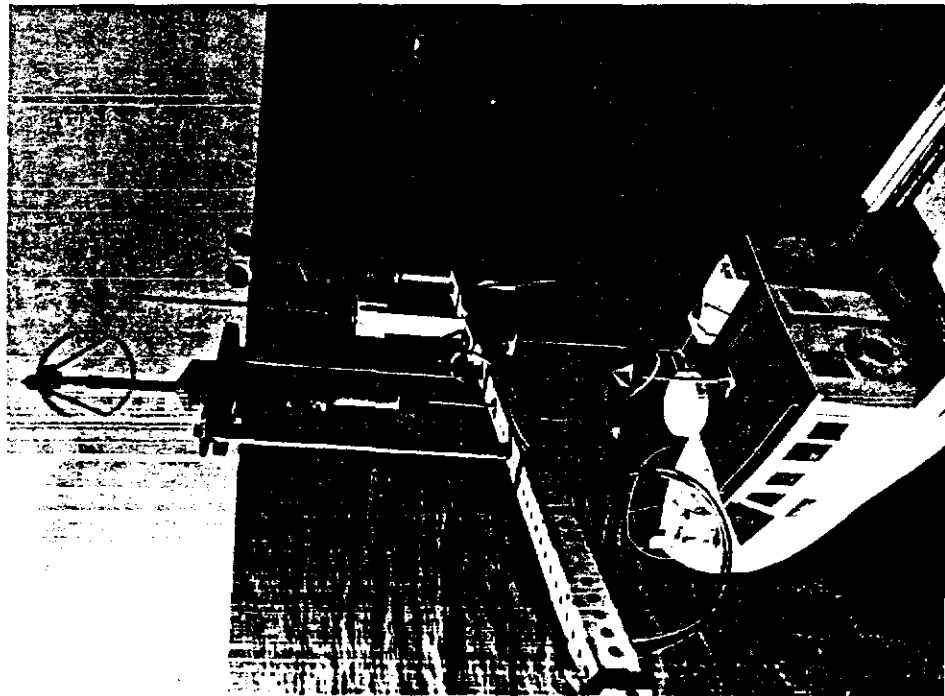


Fig. 3. A view of the gimbal-mounted bivane and single sensor hot-wire humidity sensor directional

were measured by an induction-type bivane designed and fabricated at Brookhaven National Laboratory (SethuRaman and Tuthill, 1978). Errors in the momentum flux due to the motion of the buoy are estimated to be about 1% of the true value (Pon 1968).

Capacitance-type wave staff was used to measure high-frequency wave-height variations. It consisted of a square-wave oscillator whose frequency varied linearly with a small change in sensor capacitance. This square-wave frequency was integrated and smoothed in an operational amplifier to yield a voltage output proportional to a change in sensor capacitance that exists between the water and the center conductor of an insulated wire 4 mm in diameter.

Mean wind speed and temperature were recorded continuously at the buoy, using a cup anemometer and thermistor, respectively. Mean wind directions were obtained with a directional vane at the beach. Sea surface temperature was measured with a thermistor attached to a float during the experiments. Power was supplied by a 12 V battery and the analog outputs from the hot wire, bivane and wave staff were recorded on magnetic tapes. Each ten minutes of data were then digitized at a sampling rate of sixteen per second and analyzed.

### 3. Relative Motion of the Sea Surface

The drag conditions at the air-sea interface differ from those existing over a rigid surface due to the mobility of the surface. In large deep bodies of water, wind-drive surface waves have different phase velocities. There are also conditions when the waves propagate in a direction opposite to that of the wind. Restricting our analysis to cases when the waves propagate in the same direction as the wind, the drag at the sea surface should depend on the relative motion between the air and the waves at different phase velocities. Waves propagating with a phase velocity equal to or greater than that of the mean wind speed at the level of waves would contribute little to the momentum flux into the ocean. Thus the high frequency gravity waves which move relatively slower than the wind act as mobile roughness elements causing the surface drag. A measure of their height thus corresponds to the characteristic height of the sea surface roughness,  $h_s$ .

Assuming the sea surface to be represented by two-dimensional waves of amplitude,  $a$ , propagating with a phase velocity,  $c$ , Kitaigorodskii (1973) has developed a relationship for  $h_s$  as

$$h_s = 2 \left[ \int_0^{\infty} S(\omega) \exp \left[ \frac{-2kg}{u_*} \right]^{1/2} d\omega \right] \quad (1)$$

where  $\omega$  is the radian frequency of the wave spectra,  $S(\omega)$  is the spectral density of the wave height variations,  $g$  is the gravitational acceleration,  $k$  is von Karman constant and  $u_*$  is friction velocity. This simple model has several assumptions, but

with higher frequencies to the characteristic roughness height are significant. The wind-driven wave spectra usually indicated a dominant frequency  $n_0$  in the range of 0.1 to 0.2 Hz at this site (SethuRaman, 1978a).

If the equilibrium wave spectrum given by Phillips (1966)

$$S(\omega) = \beta g^2 \omega^{-5} \quad (2)$$

where  $\beta$ , a constant, is substituted in Equation 1, the characteristic height of surface roughness will be given by

$$h_s = \frac{\sqrt{3\beta} u_*^2}{2k^2 g} \quad (3)$$

With  $k = 0.35$  and  $\beta = 0.0117$  (Phillips, 1966),

$$h_s = 0.764 \frac{u_*^2}{g} \quad (4)$$

Charnock (1955) gave a relationship based on dimensional arguments for roughness length  $z_0$  as  $\alpha u_*^2/g$  where  $\alpha$  is a constant. For an aerodynamically rough surface,  $h_s/z_0$  seems to assume a unique value of about 30 based on the classical experiments by Nikuradse in 1933 (Schlichting, 1968). For  $z_0$ , Equation 4 then becomes,

$$z_0 = 0.026 u_*^2/g \quad (5)$$

Wu (1969) found the value of  $\alpha$  to be about 0.02 and Garratt (1977) quotes a value of 0.014. Previous measurements showed the variability of  $\alpha$  with the degree of roughness of the surface and indicated a mean value of about 0.03 (SethuRaman and Raynor, 1975). Kitaigorodskii (1968) and Kitaigorodskii and Zaslavskii (1974) also found  $\alpha$  to vary with surface roughness.

#### 4. Results

##### 4.1 AERODYNAMIC ROUGHNESS OF THE SEA SURFACE

On the basis of a qualitative analysis of the wall region of the turbulent boundary layer above a solid surface, the dimensionless parameters that determine  $C_D$  can be expressed as shown below:

$$C_D = C_D \left( \frac{z}{h_s}, \frac{z}{L}, \frac{h_s}{\delta_v} \right) \quad (6)$$

where  $z$  is the height above the surface,  $h_s$  is the characteristic height of the sea surface and  $\delta_v$  is the thickness of the viscous sublayer. For  $z \ll L$ , where  $L$  is Monin-Obukhov length characterizing atmospheric stability,

$$C_D = C_D \left( \frac{z}{h_s}, \frac{h_s}{\delta_v} \right) \quad (7)$$

This condition of  $z \ll L$  could be realized either during neutral atmospheric conditions or within a few meters above the earth's surface for moderate diabatic conditions. The ratio  $\nu/u_*$ , where  $\nu$  is the kinematic viscosity and  $u_*$  the friction velocity, is of the same order of magnitude as  $\delta_v$ , and the constant of proportionality for smooth flow is about 0.1 (Schlichting, 1968). Thus for  $z \gg h_s$ ,

$$C_D = C_D \left( \frac{h_s}{\delta_v} \right) = C_D \left( \frac{u_* z_0}{\nu} \right)$$

where  $z_0$  is the roughness length that could be estimated from a knowledge of  $h_s$ . For the conditions when  $z_0 \gg \nu/u_*$ , the protrusions of roughness near the surface extend beyond the viscous sublayer, the resistance to flow is mainly due to form drag, and the surface is aerodynamically rough. When  $z_0$  is the same order of magnitude as  $\nu/u_*$  protrusions extend partly outside the sublayer leading to a transition or moderate rough regime. When  $z_0 \ll \nu/u_*$ , the size of roughness is so small that all protrusions are contained within the sublayer and the viscous drag is predominant.

Surface drag coefficient  $C_D$  was obtained from momentum flux measurements

$$C_{D_s} = (-\overline{u'w'})/u_{s8}$$

where  $u'$  and  $w'$  are the longitudinal and vertical velocity fluctuations and  $u_{s8}$  the mean wind speed at 8 m. Variation of  $C_{D_s}$  is shown as a function of  $(u_* h_s)/\nu$  in Figure

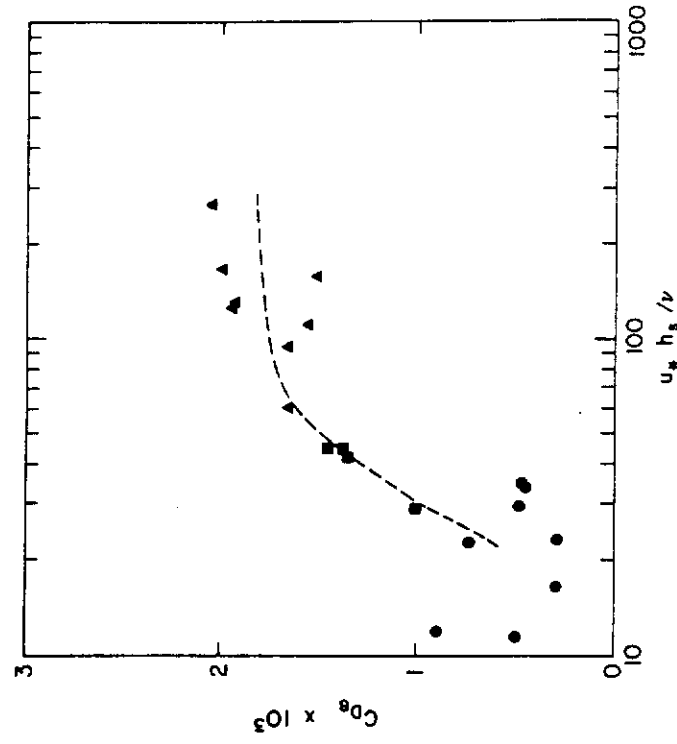


Fig. 4. Variation of  $C_{D_s}$  with  $u_* h_s/\nu$ . Aerodynamically smooth conditions are represented by circles, moderately rough by squares, and fully rough by triangles.

... values of  $C_D$  are consistent with those reported in the literature (Garratt, 1977). Characteristic height  $h_s$  was obtained from wave height spectra based on Equation 1. There is some scatter in the data points due to the indirect method of estimating  $h_s$ , but the influence of the aerodynamic roughness of the sea surface on drag coefficient seems to be clear, as shown by the dotted line drawn by eye. For  $u_* h_s / \nu < 20$ , the sea surface is aerodynamically smooth with a mean  $C_D$  of about 0.5. For  $u_* h_s / \nu > 75$ , the surface is fully rough with a mean  $C_D$  of about 1.75; the transition region of moderately rough conditions is for  $20 < u_* h_s / \nu < 75$ .

For an aerodynamically rough surface,  $z_0/h_s \approx 30$ . Hence if  $h_s$  is known,  $z_0$  can be estimated as  $h_s/30$ . Variation of  $C_D$  with  $u_* z_0 / \nu$  ( $z_0$  obtained from this ratio) indicates the surface to be smooth for  $u_* z_0 / \nu < 0.6$  and fully rough for  $u_* z_0 / \nu > 2.5$ . Similar results were previously found with  $u_*$  and  $z_0$  obtained from mean wind profiles observed at the beach with onshore flows (SethuRaman and Raynor, 1975).

#### 4.2. $C_D$ AND $u_*$ AS A FUNCTION OF $\bar{u}$

$C_D$  is plotted against  $\bar{u}$  in Figure 5. There is no systematic variation of  $C_D$  with wind speed. The results pertaining to different roughness categories defined by the respective values of  $u_* h_s / \nu$  tend to have different means. The present data indicate that the flow changes into the next higher roughness category as the wind speed increases although there is no definite wind speed at which one could claim that the flow becomes modified. There seems to be a division between the different surface roughness categories in Figure 5, whereas in practice no such clear separation is to be expected. As mentioned previously, the observations were made on four consecutive

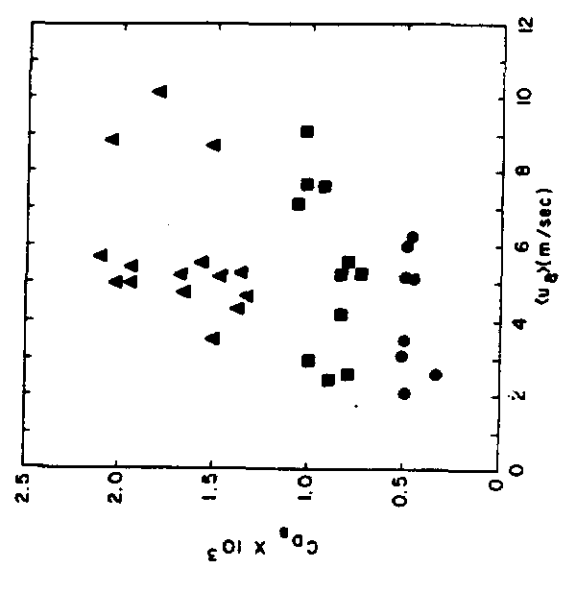


Fig. 5. Variation of surface drag coefficient  $C_D$  with mean wind speed  $\bar{u}$ . Triangles represent aerodynamically rough, squares moderately rough, and circles smooth surface conditions.

... which Hevel... was... existed on two days and partly developed wave conditions on the other two days, thus causing different wave ages be present (SethuRaman, 1978b). Previous wind profile analysis that consisted observations over several months also indicated  $C_D$  to be independent of (SethuRaman and Raynor, 1975). Kraus (1967) and Weiler and Burling (1971) among others also found  $C_D$  to be invariant with wind speed. Garratt (1977) in recent review paper on  $C_D$  gives a linear and a power law relationship for  $C_D$  a function of  $\bar{u}$  and states that the effect of wave age determined by fetch and wind duration were obscured in the experimental scatter. Most of the  $C_D$  observations reported in the literature do not have simultaneous wave data or other observations from which wave age can be estimated. An attempt will be made in this paper in later section to correlate surface drag with  $h_s$  estimated from wave height variations.

Variation of  $u_*$  with  $\bar{u}$  is shown in Figure 6. Data points were again segregated according to roughness classification as deduced from  $u_* z_0 / \nu$  values. The lines corresponding to the three different roughness classifications in Figure 6 were obtained based on profile measurements (SethuRaman and Raynor, 1975) and have the following relationships:

Smooth

$$u_* = 0.025 \bar{u}_6$$

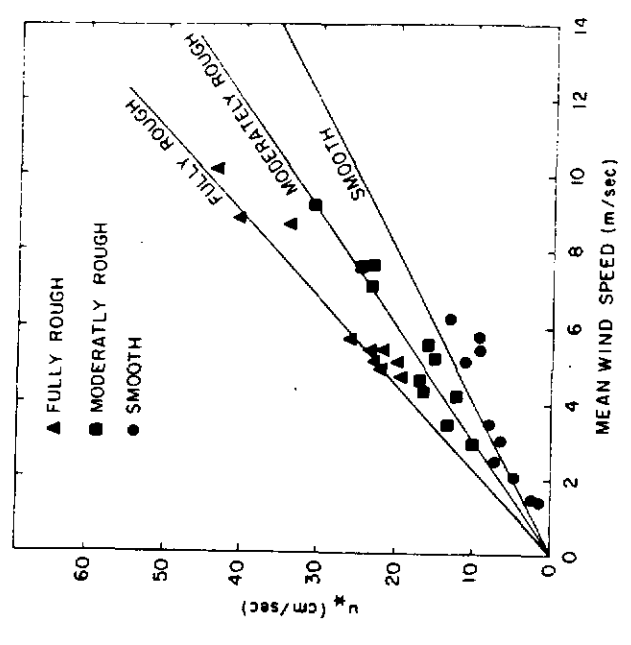


Fig. 6. Variation of friction velocity  $u_*$  with the mean wind speed  $\bar{u}$ . Symbols are the same as those in Figure 5. The lines correspond to the previous results derived from mean wind profile analysis (SethuRaman and Raynor, 1975).

Moderately rough

$$u_* = 0.033 \bar{u}_6 \quad (11)$$

Fully rough

$$u_* = 0.045 \bar{u}_6 \quad (12)$$

where  $\bar{u}_6$  is the mean wind speed at a height of 6 m. Present data agree reasonably well with these relations obtained from wind profile observations.

#### 4.3. VARIATION OF $u_*^2$ WITH $h_s$

For fully rough sea conditions, Charnock (1955) obtained a relationship of the form

$$\frac{gz_0}{u_*^2} = \alpha, \quad \alpha \text{ a constant.} \quad (13)$$

More recently Kitaigorodskii (1968) and Kitaigorodskii and Zaslavskii (1974) found  $\alpha$  to be variable for transitional flow such that

$$\frac{gz_0}{u_*^2} = f(c_0/u_*) \quad (14)$$

where  $c_0$  is the phase velocity of the dominant wave and the ratio  $c_0/u_*$  describes the degree of wave development. SethuRaman and Raynor (1975) found the value of  $\alpha$  to vary between 0.016 to 0.072 depending on the degree of roughness or wave age and obtained a mean value of about 0.034 for rough conditions. Assuming  $h_s = 30z_0$  (Schlichting, 1968), a relationship between  $u_*^2$  and  $h_s$  can be obtained as

$$u_*^2 = \frac{gh_s}{30\alpha}. \quad (15)$$

The data reported here were taken during transitional and equilibrium conditions. Variation of  $u_*^2$  with  $h_s$  is shown in Figure 7. Also shown is the relationship given by Equation 15 for  $\alpha$  equal to 0.034. This relation is also roughly equivalent to Equation 4 obtained from the relative motion approximation and Phillips' wave relationship. Data have been segregated in Figure 7 according to equilibrium or transitional conditions estimated from the variations of mean wind speeds and directions. Equilibrium conditions correspond to fully developed waves and the transitional conditions to partially developed waves. The observations in Figure 7 seem to agree roughly with the empirical relationship given by Equation 15, considering the uncertainties in the estimated values of  $\alpha$  and  $h_s/z_0$ . The transitional conditions seem to cause more drag as compared with equilibrium or fully developed wave conditions.

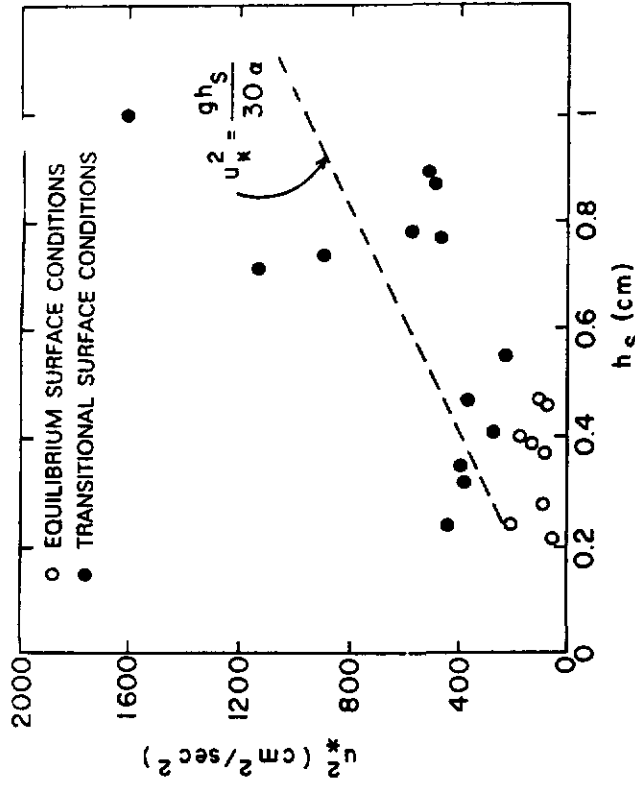


Fig. 7. Variation of  $u_*^2$  with  $h_s$ . An empirical relationship between the two parameters is also shown. Solid circles represent transitional surface conditions and the open circles equilibrium surface conditions.

#### 4.4. VARIATION OF SURFACE SHEAR STRESS WITH $h_s$

On purely empirical grounds, surface shear stress  $\tau$  can be derived from Equation 4 as

$$\tau = \frac{\rho gh_s}{30\alpha}$$

This gives a linear variation of  $\tau$  with  $h_s$ . With  $\alpha = 0.034$  as an average value for rough conditions,

$$\tau = 1.24 h_s$$

where  $\tau$  is the surface shear stress in dyne  $\text{cm}^{-2}$  and  $h_s$  is the characteristic height of the sea surface in cm. Values of surface shear stress for the observations have been plotted against  $h_s$  in Figure 8. Linear regression for all observations was found

$$\tau = 1.37 h_s - 0.02$$

with a correlation coefficient of 0.72. Since Equation 16 may not be applicable to smooth sea surface, another linear regression relationship was computed for values of  $\tau > 0.2$  dyne  $\text{cm}^{-2}$  which was assumed to be the upper limit for a smooth sea surface. The relationship for the observations of  $\tau > 0.2$  was found to be

$$\tau = 1.21 h_s - 0.04$$

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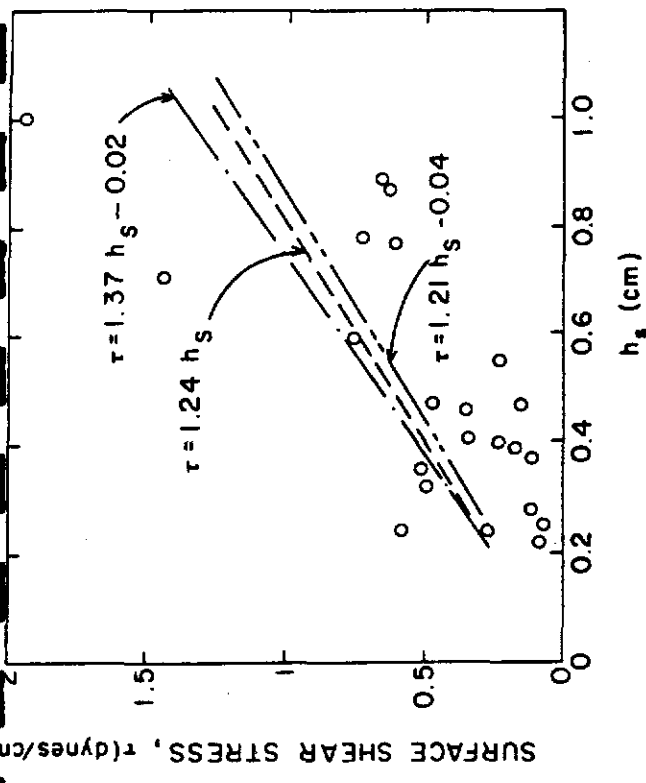


Fig. 8. Surface shear stress values as a function of  $h_s$ . A line based on empirical relationship,  $\tau = 1.24 h_s$ , shown.  $\tau = 1.37 h_s - 0.02$  corresponds to all observations.  $\tau = 1.21 h_s - 0.04$  corresponds to observations with  $\tau > 0.2$  dyne  $cm^{-2}$ .

with a correlation coefficient of 0.65. Linear relationships based on Equation 17 to are also shown in Figure 8. Although there is some scatter in the data, there is good agreement between the expected and the actual relations. More observations under a variety of wave conditions will be needed to improve the accuracy of these relationships. The characteristic height of sea surface  $h_s$  will depend on  $c_0/u_*$ , a ratio that defines wave age. The value of  $c_0/u_*$  in turn depends on wind, its duration and fetch.

5. Conclusions

Drag coefficients ( $C_D$ ) derived from momentum flux measurements over a stable, moored buoy show the existence of three aerodynamic roughness categories over water - smooth, moderately rough and fully rough. An attempt is made to determine a characteristic roughness height of the sea surface,  $h_s$ ; the high-frequency wave heights with phase velocities less than the fetches for the experiments varied from 5 to 100 m. The  $u_*^2$  was found to vary with  $h_s$  for an aerodynamically rough sea surface and approximately satisfied a relationship of the form,  $u_*^2 = gh_s/30\alpha$ . The fetch  $\tau$  was found to vary with  $h_s$  with an approximately linear relationship.