A COMPARISON OF A EULERIAN AND A LAGRANGIAN TIME SCALE FOR OVER-WATER ATMOSPHERIC FLOWS DURING STABLE CONDITIONS*

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Abstract. Lagrangian integral time scales were calculated from crosswind concentration distributions of oil-fog smoke released from a continuous point source over the ocean during stable atmospheric conditions assuming an exponential correlation function. Variance of the lateral velocity fluctuations, σ_v^2, and the energy dissipation rate, ε, were obtained from simultaneous Eulerian measurements at the beach. An Eulerian energy dissipation scale defined as σ_v^2/ε was then computed. The ratio of the Lagrangian integral scale to the Eulerian energy dissipation scale was found to be close to 1. This ratio was also estimated to be 1 based on physical and dimensional considerations regarding the cascade of energy. Length scales for longitudinal, lateral and vertical directions were interpreted with a model based on similarity considerations applicable for over-water atmospheric flows.

1. Introduction

This paper examines the interaction of turbulence and diffusion scales over water. It is found that the measured scale relationships must be interpreted taking into account the differences in physical behaviour observed for an over-water flow as against a typical over-land flow. The atmospheric diffusion is Lagrangian in nature but Lagrangian measurements of the turbulence associated with their dispersion are difficult to obtain; fixed point Eulerian measurements of turbulence are relatively simpler to find. Hence, several theoretical and experimental studies have been made to relate Eulerian and Lagrangian properties, especially time scales (Gifford, 1967). A simple scale relationship between Eulerian and Lagrangian frames of reference was suggested by Gifford (1955) based on their spectral peaks. Assuming similarity of correlation functions, Hay and Pasquill (1959) hypothesized that $R_L(\xi) = R_E(t)$ with $\xi = \beta t$ where $R_L(\xi)$ and $R_E(t)$ are the Lagrangian and Eulerian correlation functions, respectively. Hay and Pasquill (1959), Barad (1959), Panofsky (1962), Thompson (1965, 1966) and Haugen (1966), among others, have evaluated β based on the comparisons of the diffusion of a mass tracer with Eulerian measurements of wind fluctuations. Gifford (1955) and Angell (1964, 1971) estimated β using balloons for Lagrangian measurements. The values of β obtained by different investigations varied from about 1 to 8; however, an increase of β with i^{-1}, where i is the intensity of turbulence or turbulence level given by the ratio of the standard deviation of the fluctuations to the mean wind speed, was

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apparent (Philips, 1967). Values of βi vary from 0.35 to 0.8 (Pasquill, 1974). Taylor's 'frozen eddy' hypothesis is commonly used in computing the integral time scales of the fluctuations measured at a fixed point. The assumption is that the spatial pattern of turbulence is moved across the fixed point virtually unchanged at the mean wind speed, $\bar{u}$ so that the time scale is given by $l/\bar{u}$ where l is the Eulerian integral length scale.

The purpose of this paper is to demonstrate the effectiveness of an Eulerian energy dissipation time scale, τ_ε, expressed as

$$(\tau_\varepsilon)_x = \sigma_u^2/\varepsilon, \qquad (\tau_\varepsilon)_y = \sigma_v^2/\varepsilon, \qquad (\tau_\varepsilon)_z = \sigma_w^2/\varepsilon \tag{1}$$

in estimating the Lagrangian time scale τ_L where ε is the energy dissipation rate and σ_u, σ_v and σ_w are the standard deviations of the velocity fluctuations for the longitudinal (x), lateral (y) and vertical (z) directions, respectively. Combining with a 'coupling eddy' concept suggested by Inoue (1959), where particles may be considered to move with a fixed eddy pattern at a characteristic speed σ_k, the length scale would be similar to that derived by Tennekes and Lumley (1972).

2. Measurements

The Eulerian and Lagrangian observations here pertain to the over-water diffusion experiments conducted off the south shore of Long Island (Raynor *et al.*, 1975). Oil fog smoke was used as a tracer released at a height of about 6 m from a boat anchored offshore. Concentration measurements were made with another boat or a vehicle on the beach using optical densitometers, similar in operation to integrating nephelometers. Several passes were made at each distance to obtain representative statistical parameters of the meandering plume. One of the parameters estimated from the concentration measurements across the plume was the lateral standard deviation σ_y. The travel distance from the source to the line of measurements varied from 0.5 to 6 km.

Meteorological measurements (SethuRaman *et al.*, 1974) were made on the beach at heights of 16 or 24 m with a bivane except for two experiments in which a directional vane was used. Hot-film and hot-wire sensors were occasionally used. The frequency response of the bivane was found to be about 2 Hz (SethuRaman and Brown, 1976), enough to obtain a reliable ε. The sampling time for the meteorological measurements was about the same as the time elapsed in making several traverses across the plume at each downwind distance. The wind fluctuations were recorded in analog form and digitized after passing through a low-pass active RC filter. Conventional spectrum analysis (Blackman and Tukey, 1959) was then performed.

3. Eulerian and Lagrangian Time Scales

The energy dissipation rate ε_y was obtained from the inertial subrange of the

v-spectra with the following relationship,

$$S_v(K) = \alpha \varepsilon_y^{2/3} K^{-5/3} \tag{2}$$

where α is Kolmogorov's universal constant with a value of 0.38. The energy dissipation time scale for the lateral y direction was then obtained as the ratio of the available kinetic energy in the y direction to the rate of its dissipation expressed as,

$$\tau_{\varepsilon_y} = \sigma_v^2/\varepsilon. \tag{3}$$

τ_{ε_y} for the experiments reported here ranged from 28 to 300 s for onshore flows.

To obtain τ_L from oil-fog measurements, an assumption of the shape of the Lagrangian correlation function was necessary. An exponential correlation function of the form

$$R_L(t') = \exp(-t/\tau_L) \tag{4}$$

was used.

Draxler (1976) found that this form of correlation function was satisfactory for downwind distances ranging from 1 to 25 km for several overland experiments. For a homogeneous and stationary Eulerian field, and identical Eulerian-Lagrangian velocity distributions, the lateral standard deviation σ_y of the material released from a continuous point source may be expressed as

$$\sigma_y^2 = 2\sigma_v^2 \int_0^t (t-t')R_L(t')\,dt' \tag{5}$$

where t is the travel time of the plume or $x/\bar{u}$.

Substituting Equation (4) in Equation (5),

$$\sigma_y^2 = 2\sigma_v^2 \tau_{L_y}^2 \{t/\tau_{L_y} - 1 + \exp(-t/\tau_{L_y})\}. \tag{6}$$

The lateral standard deviation σ_y was computed from the cross-wind concentration distribution of oil-fog smoke by summing over all traverses at any particular downwind distance (Raynor *et al.*, 1975). Since σ_y, σ_v and t are known from actual observations, τ_{L_y} can be found using Equation (6). Then the ratio $(\tau_L/\tau_\varepsilon)_y$ was computed with τ_{ε_y} obtained from Equation (3). These ratios for different experiments are shown in Figure 1 as a function of lateral turbulence level $\sigma_v/\bar{u}$. The experiments reported here were conducted between 1972 and 1975. The observations were over water for onshore winds except for one case when they were offshore.

As can be seen from Figure 1, the ratios of $(\tau_L/\tau_\varepsilon)_y$ are close to 1.0 for $0.04 < \sigma_v/\bar{u} < 0.19$ (all the experiments). The geometric mean of the ratios was found to be 1.03.

The reason for the closeness of this mean ratio to one can be explained with the following physical and dimensional arguments. In a stationary condition with the rate of dissipation ε equal to the rate of feeding of turbulent energy, the total

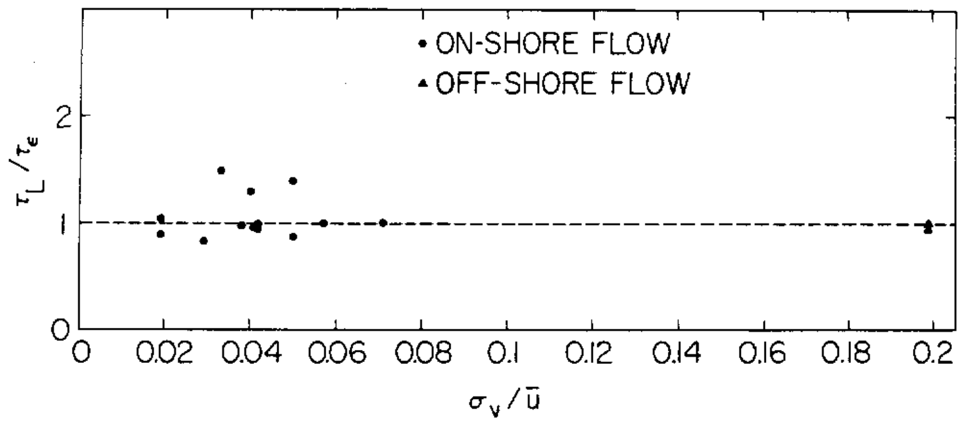

Fig. 1. The variation of the ratio of the integral Lagrangian time scales to the energy dissipation time scale, τ_L/τ_ε, with the lateral intensity of turbulence, $\sigma_v/\bar{u}$.

energy can be expected to be related to ε in accordance with the scale of turbulence (Hinze, 1959). The larger this scale, the longer will be the possible wave-number range over which the down-scale cascade of turbulent energy occurs before viscous dissipation sets in and hence the smaller will be ε; in other words,

$$\frac{\sigma_w^3}{\Lambda_z}=\frac{\sigma_u^3}{\Lambda_x}=\frac{\sigma_v^3}{\Lambda_y}=\varepsilon \tag{7}$$

where Λ_x, Λ_y, and Λ_z are the Lagrangian length scales in x, y, and z directions, respectively. If the flow is assumed to move past the fixed point at a characteristic eddy velocity σ_u in the longitudinal direction, σ_v in the lateral direction, and σ_w in the vertical direction, the Lagrangian time scales will be

$$\tau_{L_x}=\sigma_u^2/\varepsilon, \qquad \tau_{L_y}=\sigma_v^2/\varepsilon \quad \text{and} \quad \tau_{L_z}=\sigma_w^2/\varepsilon\,. \tag{8}$$

the same as the Eulerian energy dissipation scale given in Equation (1). This might be the reason for a mean ratio close to 1 in Figure 1 (with individual values varying from 0.8 to 1.5).

4. Estimation of Scale Relationships for Near-Neutral Stability Conditions

In this section, a simple model applicable for near-neutral conditions is used to gain some insight into the scale relationships in the three directions over water. In the coupling eddy concept, the flow is assumed to move past the fixed point at a characteristic eddy velocity given by the standard deviation of velocities in different directions. The mixing-length concept in the vertical direction is a special case of the coupling eddy concept where the characteristic length scale of the eddy in the vertical direction, Λ_z can be taken as kz (Inoue, 1959) where k is von Karman's constant. The arguments given below are based on a simple model *applicable for near-neutral conditions* to gain some insight into the scale relationships.

The atmospheric variables of importance in determining the internal structure of the flow are the buoyancy term $(g\overline{w'\theta'})/\theta$ (where g is the gravitational acceleration, w' is the vertical velocity fluctuation, θ' is the temperature fluctuation and θ is the mean temperature), molecular kinematic viscosity ν, friction velocity u_* and height z. At the high Reynolds numbers found in the atmosphere, for $z \gg z_0$ where z_0 is the roughness length, molecular terms can be neglected. Thus, in near-neutral conditions, all dimensionless groups such as σ_u/u_*, σ_v/u_*, σ_w/u_*, $(z/u_*)\,\mathrm{d}\bar{u}/\mathrm{d}z$, etc. must be constant and independent of height (Ellison, 1957). For conditions departing appreciably from neutrality, the Monin–Obukhov length L given by $u_*^3\theta/(g\overline{w'\theta'})$ becomes of importance. For moderately diabatic conditions or for $z \ll L$, it is reasonable to expect that the non-dimensional groups σ_u/u_*, σ_v/u_* and σ_w/u_* would still be fairly constant at least over water with long fetches so that

$$\sigma_u = au_* \tag{9}$$

$$\sigma_v = bu_* \tag{10}$$

$$\sigma_w = cu_* \,. \tag{11}$$

For adiabatic atmospheric conditions, the Lagrangian length scales Λ_x, Λ_y and Λ_z in the x, y, and z directions, respectively, can be assumed to be

$$\Lambda_x = a'z \tag{12}$$

$$\Lambda_y = b'z \tag{13}$$

$$\Lambda_z = c'z \tag{14}$$

with some reservations regarding the heights considered. The assumption that the scale of vertical velocities varies with height is well documented with observations over land. The lateral velocity spectrum *over land* appears not to obey the similarity theory for neutral and unstable conditions (Lumley and Panofsky, 1964) for the following reasons: (i) Increasing instability greatly increases the low-frequency portions of the spectrum leaving the high-frequency parts relatively unaffected; (ii) The high-frequency portion of the spectrum is dependent on roughness; and (iii) The lateral spectrum is influenced by terrain. Over-water atmospheric flow is different from over-land flow for several reasons. The mechanical roughness is several orders of magnitude smaller than that over land and the terrain is flat and homogeneous for all practical purposes. Hence the main reasons that would lead to a modified lateral velocity spectrum at different heights are absent for over-water atmospheric flow making it reasonable to assume that the lateral scale has some dependence on height at least for the lowest few tens of meters for near-neutral stability. Same arguments are applicable for the longitudinal velocity spectrum with the difference that the mean wind speed affects to a certain extent the low frequencies. Studies show (Lumley and Panofsky, 1964) that the longitudinal scale slowly increases upward in the lowest few meters. From the equation governing the budget of kinetic energy in the atmosphere, it can be shown (Taylor, 1952) that for near neutral conditions,

$$\text{energy dissipation rate } \varepsilon = \frac{u_*^3}{kz}. \tag{15}$$

For diabatic conditions, ε is approximately equal to u_*^3 (1-Rf) where Rf is the flux Richardson number if the divergence of energy flux is negligible (Panofsky, 1962). For small Rf, Equation (15) is a good approximation for ε in the vertical direction.

Inoue (1959) assumed $a \simeq 3$, $b \simeq 1.5$ and $c \simeq 1$ based on available laboratory and field observations. The variation of σ_u/u_* with near-surface Richardson number Ri_4 is shown in Figure 2 for over-water atmospheric flow.

The atmospheric stability generally varied from slightly stable to inversion conditions with the range of turbulence level $\sigma_u/\bar{u}$ between 0.04 and 0.1 and the near-surface gradient Richardson number Ri varying from about 0 to 0.15. The variation of σ_v/u_* and σ_w/u_* with Ri_4 are shown in Figures 3 and 4, respectively.

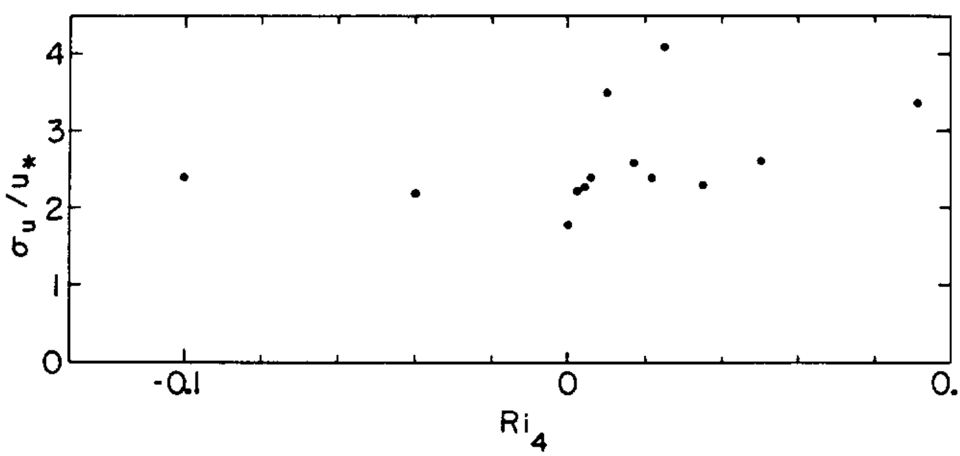

Fig. 2. The variation of σ_u/u_* with Ri_4.

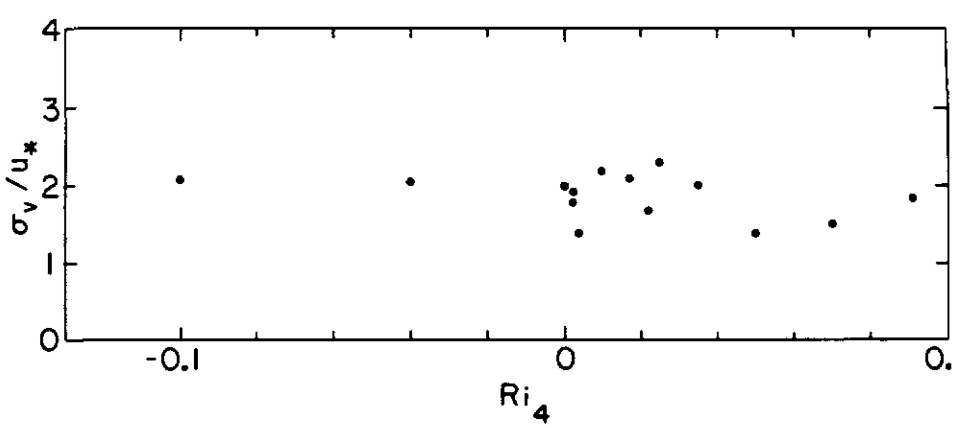

Fig. 3. The variation of σ_v/u_* with Ri_4.

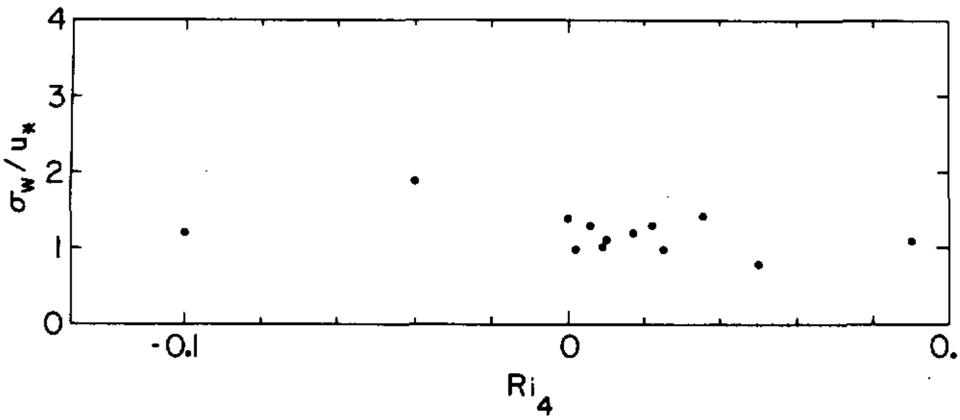

Fig. 4. The variation of σ_w/u_* with Ri_4.

There is some scatter but no definite trend with Richardson number. A value of about 0.1 for $\sigma_u/\bar{u}$ would correspond approximately to near-neutral conditions over water. The variation of σ_w/u_* with atmospheric stability has been extensively studied. A value of 1.35 for σ_w/u_* is often found during near-neutral conditions over water (Miyake *et al.*, 1970; Pond *et al.*, 1971) and a value of 1.2 over land (Monin and Yaglom, 1971). Danielson *et al.* (1974) have suggested an empirical formulation for σ_w/u_* varying with z/L based on profile relationships as below:

$$\sigma_w/u_* = 1.3[(kz/u_*)(d\bar{u}/dz) - 2.5(z/L)]^{1/3}. \quad (16)$$

Most of the observations for over-water flow on which Equation (16) was based were within a z/L of ± 0.5 and showed a considerable scatter between 1 and 1.5. On the other hand, over-land values seemed to agree better with Equation (16). Hence, a constant value of σ_w/u_* for stable conditions seems to be appropriate for over-water flow.

Table I shows the mean and standard deviations of the ratios of r.m.s. velocity fluctuations to u_*. Pasquill (1974) quotes over-land values of σ_u/u_* in the range 2.1 to 2.9.

TABLE I

Ratios of σ_k/u_* for $0 < Ri_4 < 0.15$

Ratio	No. of observations	Mean	Standard Deviation
σ_u/u_*	41	2.6	0.7
σ_v/u_*	41	2.0	0.8
σ_w/u_*	41	1.4	0.5

To examine the relative magnitudes of the Lagrangian length scales, Equation (7) is used in combination with Equations (9) to (15). By substituting the mean values for *a*, *b*, and *c* obtained during stable conditions from Table I, the mean values for

a', b', and c' happen to be 7.0, 3.2 and 1.1, respectively. Von Karman's constant k was assumed to be 0.4. These values indicate the eddy structure to be elongated in the longitudinal direction as one would expect to occur during stable conditions.

5. Conclusions

A new Eulerian time scale based on the ratio between the variance of the velocity fluctuations and the energy dissipation scale is proposed. This time scale is found to be about the same as the Lagrangian time scale obtained from cross-wind concentration measurements of oil-fog smoke released from a continuous point source over water during stable conditions. Dimensional and physical arguments regarding the cascade of energy in combination with a coupling eddy assumption also yield the same results.

An estimation of the relative values of length scales over water was made based on certain similarity assumptions for near-neutral conditions; longitudinal and lateral scales were found to be on the average about six times and three times the vertical scale, respectively, indicating an elongated eddy structure.

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