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A simple nonlinear model for shelf air-sea interaction

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Abstract. A low-order, coupled atmosphere-ocean model is developed for studying air-sea interactions over the continental shelf off the east U.S. coast. Maximum simplification is used to derive the model equations, yet several key physical processes essential to the continental shelf air-sea interaction are retained. Numerical solutions for the fully coupled system, and analytical solutions for the steady state are analyzed.

Key words: low-order model, air-sea coupling, Gulf Stream, Ekman layer

1. Introduction

Understanding the variability of sea surface temperature (SST) and ocean currents over the continental shelf off the east coast of the United States is important for a number of reasons. Meteorologically, oceanic frontal features over the shelf could enhance the local sea-to-air heat and moisture fluxes which often favor the formation of coastal atmospheric fronts, mesoscale jets and cyclones (Hsu, 1984; Huang and Raman, 1992; Doyle and Warner, 1990). These coastal weather systems are of great interest because of their importance to the development of severe coastal storms which are some times disastrous to the coastal community. From oceanographic point of view, shelf waters off the east coast is a buffer or pathway between the Gulf Stream and the coastal estuaries. Oceanic frontal features and currents greatly influence the cross-shelf exchange of water mass and biota between the Gulf Stream and the estuaries (Pietrafesa et al., 1985; Lee et al., 1989).

In principle, a coupled system linking a sophisticated regional-scale atmospheric model and an advanced three-dimensional coastal ocean model is needed to realistically simulate the coastal sea-air system. However, enormous computing power and effort to collect data for model initialization and validation are needed to execute such complex coupled models. Thus, simplified models are often more attractive than complicated models as a research tool. Low-order models are the simplest forms of simple models which can be easily solved numerically on small work stations and some times even analytically trackable.

A low-order model is generally formed by truncating the continuous forms of model equations into a series of normal modes or discretizing the equations in a network of grids, but retaining only the first a few modes or a small number of grids. Low-order models are widely used in the study of nonlinear chaotic systems. The often cited example of low-order model for geophysical fluid dynamics is the Lorenz system (Lorenz, 1963). It describes a single cell of Benard convection, heated from below. Another acclaimed low-order model is that of Charney and Devore (1979) for investigating the dynamics of persistent mid-latitude atmospheric flow patterns. Low-order models have also been widely used in air-sea interaction studies in the tropics. Vallis (1986) formed a low-order model of El Nino and Southern Oscillation, a phenomenon associated with anomalous warm SST occurring in the tropical Pacific every 2 to 5 years, by simplifying the tropical Pacific as a two-point system. One point is located in the western Pacific and the other is in the eastern Pacific. His two-point model has captured many basic features reminiscent of the real system. A great deal of advancement in our understanding of the El Nino and Southern Oscillation phenomenon has been gained by studying the low-order behavior of the coupled tropical atmosphere-ocean system. Vallis (1990) has provided an excellent review on this subject.

In this study, we will construct a low-order model for a shelf sea-air system. We will follow the formal procedure outlined in Vallis (1990) starting from the conservation principles of momentum, mass and energy. We will retain key physical processes deemed important for the shelf sea-air system while maximum simplification will be applied to reduce the spatial dimensions.

2. Model equations

2.1 The ocean

We will first consider a low-order model consisting of six C-grid cells in a cross-shelf section (x–z plane) for shelf waters off the east coast (Fig. 1). The shelf is bounded from the west by a physical coastal barrier. Denote T_-, T and T_+ as surface layer temperatures in the inner ($x = -\delta x$), middle ($x = 0$) and outer ($x = \delta x$) shelves, respectively. Assume the interior layer temperature is also T_-, and $T_+ > T > T_-$. This assumption implies a well-mixed inner shelf, a stratified middle shelf and a more strongly stratified outer shelf. This is a typical winter condition in the Carolinas' shelves. Denote the surface cross-shore ocean currents at the inner shelf ($x = -\delta x/2$) and the outer shelf ($x = \delta x/2$) as U_- and U_+, respectively, and the vertical velocity at interface in the middle shelf ($x = 0$, $z = -\delta z$) as W. The surface current in the middle shelf can be interpolated linearly from U_- and U_+, as $0.5(U_- + U_+)$.

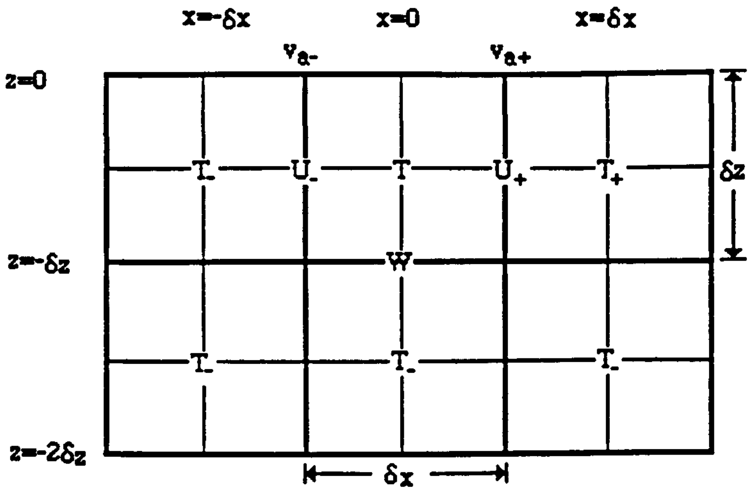

Fig. 1. Staggered C-grids used in the model.

Consider non-divergent flow in the x–z plane. The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

which can be discretized using centered difference scheme as:

$$\frac{U_+ - U_-}{\delta x} - \frac{W}{\delta z} = 0, \tag{1}$$

Assume the temperature at the mid-shelf is determined by cross-shelf advection, upwelling and air-sea heat exchange which is crudely parameterized by a Newtonian cooling effect with a time scale σ^{-1}. Let T^* be the temperature to which the mid-shelf SST would relax in the absence of motion. Then the thermodynamic equation written in continuous form is

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} - \sigma(T - T^*)$$

which can be discretized using centered difference scheme in Arakawa C-grid as

$$\frac{dT}{dt} = -U\frac{(T_+ - T_-)}{2\delta x} - W\frac{(T - T_-)}{\delta z} - \sigma(T - T^*)$$
$$= -(U_+ + U_-)\frac{(T_+ - T_-)}{4\delta x} - (U_+ - U_-)\frac{(T - T_-)}{\delta x} - \sigma(T - T^*) \quad (2)$$

The simplest assumption to make about the surface ocean current is that it is forced by surface wind stress (τ^x, τ^y). In this case, the equations governing the classical Ekman solution can be applied. Denote u, v as the ageostrophic components of the surface current. Then the momentum equations written in continuous form are

$$\frac{\partial u}{\partial t} = fv + \frac{\partial \tau_x}{\partial z} - \nu u$$

$$\frac{\partial v}{\partial t} = -fu + \frac{\partial \tau_y}{\partial z} - \nu v$$

where (τ_x, τ_y) are the shear stresses within the surface Ekman layer, and ν^{-1} is a friction time scale for the shelf waters. Since the inner shelf is well mixed, significant damping due to bottom friction occurs there. In the middle and outer shelves, friction can be caused by vertical mixing as well as bottom friction if stratification is weak. The value of ν for the inner shelf may be larger than that for the outer shelf. However, for simplicity, we will assume a constant ν for the entire shelf. The sensitivity of model results to the choice of ν will be discussed later. Denote H_- and H_+ as the Ekman layer thicknesses in the inner and the outer shelves, respectively and assume that they decrease as the vertical temperature contrast (stratification) increases,

$$H_- = D/[1 + k(T_+ - T_-)] \quad (3)$$

$$H_+ = D/[1 + k(T_+ - T_-)] \quad (4)$$

where k is a constant representing the effect of vertical stratification on the Ekman depth, and D is the unperturbed Ekman depth. In the special case of $k = 0$, $H_+ = H_-$, which represents a constant Ekman depth. Integrating the momentum equations vertically through the Ekman layer and rewrite them in finite difference form, we have

$$\frac{dU_+}{dt} = fV_+ + \frac{\tau_+^x}{D}[1 + k(T_+ - T_-)] - \nu U_+ \quad (5)$$

$$\frac{dU_-}{dt} = fV_- + \frac{\tau_-^x}{D}[1 + k(T - T_-)] - \nu U_+ \quad (6)$$

$$\frac{dV_+}{dt} = -fU_+ + \frac{\tau_+^y}{D}[1 + k(T_+ - T_-)] - \nu V_+ \tag{7}$$

$$\frac{dV_-}{dt} = -fU_- + \frac{\tau_-^y}{D}[1 + k(T - T_-)] - \nu V_-. \tag{8}$$

Denote the along-shore wind components over the inner and outer shelves as v_{a-} and v_{a+}, respectively, and the corresponding cross-shelf wind components as u_{a-}, u_{a+}. Then a linear drag relation leads to

$$\tau_+^x = ru_{a+} \tag{9}$$

$$\tau_-^x = ru_{a-} \tag{10}$$

$$\tau_+^y = rv_{a+} \tag{11}$$

$$\tau_-^y = rv_{a-} \tag{12}$$

where r is a linear drag coefficient. From (5)–(12), we may rewrite U_+ and U_- in terms of vertical temperature gradients and along-shore surface winds:

$$\frac{dU_+}{dt} = fV_+ + \frac{ru_{a+}}{D}[1 + k(T_+ - T_-)] - \nu U_+ \tag{13}$$

$$\frac{dU_-}{dt} = fV_- + \frac{ru_{a-}}{D}[1 + k(T - T_-)] - \nu U_+ \tag{14}$$

$$\frac{dV_+}{dt} = -fU_+ + \frac{rv_{a+}}{D}[1 + k(T_+ - T_-)] - \nu V_+ \tag{15}$$

$$\frac{dV_-}{dt} = -fU_- + \frac{rv_{a-}}{D}[1 + k(T - T_-)] - \nu V_-. \tag{16}$$

2.2 The atmosphere

We will assume the atmospheric surface perturbation winds over the shelf waters (u'_a, v'_a) are forced by the cross-shelf SST gradient. It is considered that the cross-shelf SST gradient induces a cross-shelf pressure gradient in the atmosphere which then drives the surface flow field. Such a consideration has often been adopted in studies of land-sea breeze type circulations in the coastal region (Hauwritz, 1947; Hsu, 1984). Thus momentum equations for the surface perturbation winds can be written as

$$\frac{du'_{a+}}{dt} = fv'_{a+} + \Pi(T_+ - T) - \mu u'_{a+} \tag{17}$$

$$\frac{du'_{a-}}{dt} = fv'_{a-} + \Pi(T - T_-) - \mu u'_{a+} \quad (18)$$

$$\frac{dv'_{a+}}{dt} = -fu'_{a+} - \mu v'_{a+} \quad (19)$$

$$\frac{dv'_{a-}}{dt} = -fu'_{a-} - \mu v'_{a-} \quad (20)$$

where Π is a constant representing the effect of cross-shelf SST gradient on the surface pressure gradient force and μ is a Rayleigh friction coefficient for surface wind. Based on the estimate of Hsu (1988) for mesoscale circulations over the Gulf Stream at a latitude of roughly 37°N, the typical values for Π and μ are $\Pi = 6.4 \cdot 10^{-5}\,\mathrm{ms^{-2}deg^{-1}}$, and $\mu = 3.2 \cdot 10^{-5}\,\mathrm{s^{-1}}$. The total cross-shelf and along-shore wind components are

$$u_{a+} = u'_{a+} + u_{am} \quad (21)$$

$$u_{a-} = u'_{a-} + u_{am} \quad (22)$$

$$v_{a+} = v'_{a+} + v_{am} \quad (23)$$

$$v_{a-} = v'_{a-} + v_{am} \quad (24)$$

where u_{am} and v_{am} represent, respectively, the cross-shelf and the along-shore components of the ambient wind.

The nine ordinary differential equations (2), (13) – (20) and diagnostic equations (21) – (24) are a closed system for nine time-dependent variables T, U_+, U_-, V_+, V_-, u_{a+}, u_{a-}, v_{a+}, and v_{a-}, if T_+ and T_- are treated as known boundary values. In this case, T_- and T_+ can be considered as the western and eastern boundary conditions which may be assumed either as constants or time-dependent parameters. For example, a time dependent T_+ may represent a varying Gulf Stream flowing along the eastern edge of the shelf. A more involved (but straight forward) development is to introduce two additional prognostic equations similar to (2) for T_+ and T_-, respectively. Then, we will need to specify cross-shelf currents at the coast (which can certainly be assumed as zero), and at the shelf edge which may be assumed as the cross-shelf current induced by Gulf Stream meanders. In this study, we will simply assume T_+ and T_- as known constants.

Since (2) is nonlinear, the equations governing the coupled shelf air-sea system form a nonlinear dynamical system which can not be solved analytically, in general. In the following section, we will analyze the steady state solutions and the transient solutions under different approximation assumptions.

3. Results

3.1 Steady state solutions

Consider first the steady state solutions by setting the left hand side of the governing equations (2) and (13) – (20) to zero.

The steady state thermodynamic equation is

$$(U_+ + U_-)\frac{(T_+ - T_-)}{4\delta x} + (U_+ - U_-)\frac{(T - T_-)}{\delta x} + \sigma(T - T^*) = 0 \qquad (25)$$

Equation (25) states the balance among cross-shelf advection, upwelling, and diabatic cooling. In the absence of Newtonian cooling effect ($\sigma = 0$), the equilibrium SST in the mid-shelf must be maintained by the onshore (offshore) advection of warm (cold) water and the upwelling of cold (downwelling of warm) water from below (above). If, in this case, there is no upwelling (e.g., no temperature stratification over the mid-shelf, or the cross-shelf current is uniform), equilibrium SST can not exist. In fact, the mean current $(U_+ + U_-)$ and the mean SST gradient $(T_+ - T_-)$ will constantly warm the mid-shelf SST which will eventually violate the physical laws and become unrealistic. Therefore, mid-shelf warming due to cross-shelf advection must be balanced either by a Newtonian cooling (air-sea heat exchange) or upwelling of cold water from below in order to reach an equilibrium solution.

The steady state momentum equations for the surface ocean currents are

$$U_+ = B[v_{a+} + (\nu/f)u_{a+}][1 + k(T_+ - T_-)] \qquad (26)$$

$$U_- = B[v_{a-} + (\nu/f)u_{a-}][1 + k(T - T_-)] \qquad (27)$$

$$V_+ = (\nu/f)U_+ - r(fD)^{-1}u_{a+}[1 + k(T_+ - T_-)] \qquad (28)$$

$$V_- = (\nu/f)U_- - r(fD)^{-1}u_{a-}[1 + k(T - T_-)] \qquad (29)$$

where $B = r/[fD(1 + \nu^2/f^2)]$ which represents the intensity of oceanic current response to atmospheric wind forcing. If $k = \nu = 0$, (26) – (29) represent the classical Ekman solutions induced by surface wind stresses over a well-mixed water body of constant depth D.

The steady state equations for the surface perturbation winds are

$$u'_{a+} = -\mu v'_{a+}/f$$

$$u'_{a-} = -\mu v'_{a-}/f$$

$$v'_{a+} = -\Pi(T_+ - T)/f + \mu u'_{a+}/f$$

$$v'_{a-} = -\Pi(T - T_-)/f + \mu u'_{a-}/f.$$

They can be rearranged as

$$u'_{a+} = \mu a(T_+ - T)/f \tag{30}$$

$$u'_{a-} = \mu a(T - T_-)/f \tag{31}$$

$$v'_{a+} = -a(T_+ - T) \tag{32}$$

$$v'_{a-} = -a(T - T_-). \tag{33}$$

Here,

$$a = \Pi[f(1 + \mu^2/f^2]^{-1} \tag{34}$$

which is a parameter representing the intensity of atmospheric response to oceanic thermal forcing. Thus, (30) – (33) represent the steady state responses of surface winds over a oceanic SST front. Hsu (1984) derived a similar relationship between the mean velocity of a sea-breeze-like circulation across an SST-front. For a sea-breeze circulation over a SST front at around 37 °N, Hsu (1988) suggests that the average value for $\mu a/f$ is approximately $2\,\mathrm{ms}^{-1}\mathrm{deg}^{-1}$.

Replacing the surface winds in (26) – (29) by cross-shelf SST gradient expressed by (30) - (33), the cross-shelf ocean currents over the outer and the inner shelves can be rewritten, respectively, as

$$U_+ = B[1 + k(T_+ - T_-)][-A(T_+ - T) + v^*] \tag{35}$$

$$U_- = B[1 + k(T - T_-)][-A(T - T_-) + v^*] \tag{36}$$

where

$$A = a(1 - \mu\nu/f^2) \tag{37}$$

$$v^* = (v_{am} + \nu f^{-1} u_{am}).$$

The algebraic equations (25), (35) and (36) are a closed set for T, U_-, and U_+.

We may substitute (35) and (36) into (25) to obtain a third order algebraic equation for the perturbation SST over the mid-shelf ($T' = T - T_-$):

$$\alpha T'^3 + \beta T'^2 + \gamma T' + c = 0 \tag{38}$$

where

$$\alpha = -BAk/\delta x$$

$$\beta = 0.75\alpha(T_+ - T_-) - B(2A - kv^*)/\delta x$$

$$\gamma = -B(T_+ + T_-)\{1.25kv^* - A[1 + k(T_+ - T_-)]\}/\delta x - \sigma$$

$$c=-0.25B(T_+-T_-)\{2v^*+(T_+-T_-)[k[v^*-A(T_+-T_-)]-A]\}/\delta x-\sigma(T_--T^*).$$

The intensity of air-sea coupling is described by the values of A and B which are determined by the thermal response coefficient Π, drag coefficient r, and the friction coefficients μ and ν. Equations (34) and (37) indicate that the coupling strength between the atmosphere and the oceans increases as the drag coefficient r and the thermal forcing parameter Π increase and decreases as the friction coefficients μ and ν increase. Thus, the net coupling strength is determined by the relative intensity of forcing to that of damping. For a fixed drag coefficient and friction coefficients, the intensity of coupling is determined by Π or its equivalence A. So we can use A as an external parameter to examine the sensitivity of the steady states to air-sea coupling strength.

The following set of parameters are used in the computations to be discussed below: $T_+ = 20\,^\circ$C, $T_- = 10\,^\circ$C, $T^* = 10\,^\circ$C, $\delta x = 30{,}000$ m, $\delta z = 20$ m, $f = 7.29 \cdot 10^{-5}\,\mathrm{s}^{-1}$, $\mu = 2 \cdot 10^{-5}\,\mathrm{s}^{-1}$, $r = 2 \cdot 10^{-5}\,\mathrm{s}^{-1}$, $\nu = 5.79 \cdot 10^{-6}\,\mathrm{s}^{-1}$, $\sigma = 1.16 \cdot 10^{-6}\,\mathrm{s}^{-1}$, $u_{am} = 0$.

The value of v_{am} and Π are respectively $-5\,\mathrm{ms}^{-1}$ and $6.4 \cdot 10^{-5}\,\mathrm{ms}^{-1}\mathrm{deg}^{-1}$, except in the sensitivity computations where their values vary. For the transient calculations, the initial value for T is $10\,^\circ$C and the initial perturbation current and wind are zero everywhere.

i) Sensitivity to coupling strength.

We will first examine the sensitivity of steady state solutions of the coupled system to the coupling strength. For simplicity, we will for the time being set $k = 0$, so the Ekman layer depth is uniform over the entire shelf.

Figure 2a shows the steady state solutions for the mid-shelf SST as a function of coupling coefficient A. In general, the value of A is positive since a higher SST induces lower surface atmospheric pressure so the atmospheric pressure gradient induced by an oceanic front must be in the opposite direction to the SST gradient. So the surface perturbation winds blow from cold water toward warm water. Fig. 2a indicates that there are two mathematical solutions for the steady states. However, only the one warmer than the interior temperature ($10\,^\circ$C) is realistic. It varies from roughly $12\,^\circ$C for $A = 0.125\,\mathrm{ms}^{-1}\mathrm{deg}^{-1}$ to $15\,^\circ$C for $A = 1.425\,\mathrm{ms}^{-1}\mathrm{deg}^{-1}$. Although the variance of the steady state SST over the mid-shelf is only $3\,^\circ$C over almost an order of magnitude range of A, we would state the dependence of mid-shelf SST on A as sensitive since only a couple of degrees of difference could cause $3-4\,\mathrm{ms}^{-1}$ of difference in along-shore wind speed. The second steady state has a value less than interior temperature T_- and becomes unbounded as A approaches 0. Thus it is purely computational, which has no counterpart in the real world.

The sensitivity of surface perturbation winds and currents to the coupling strength exhibits the same characteristics as that of mid-shelf SST. This can be expected from the fact that they are linear functions of mid-shelf SST as shown in equations (26) – (34).

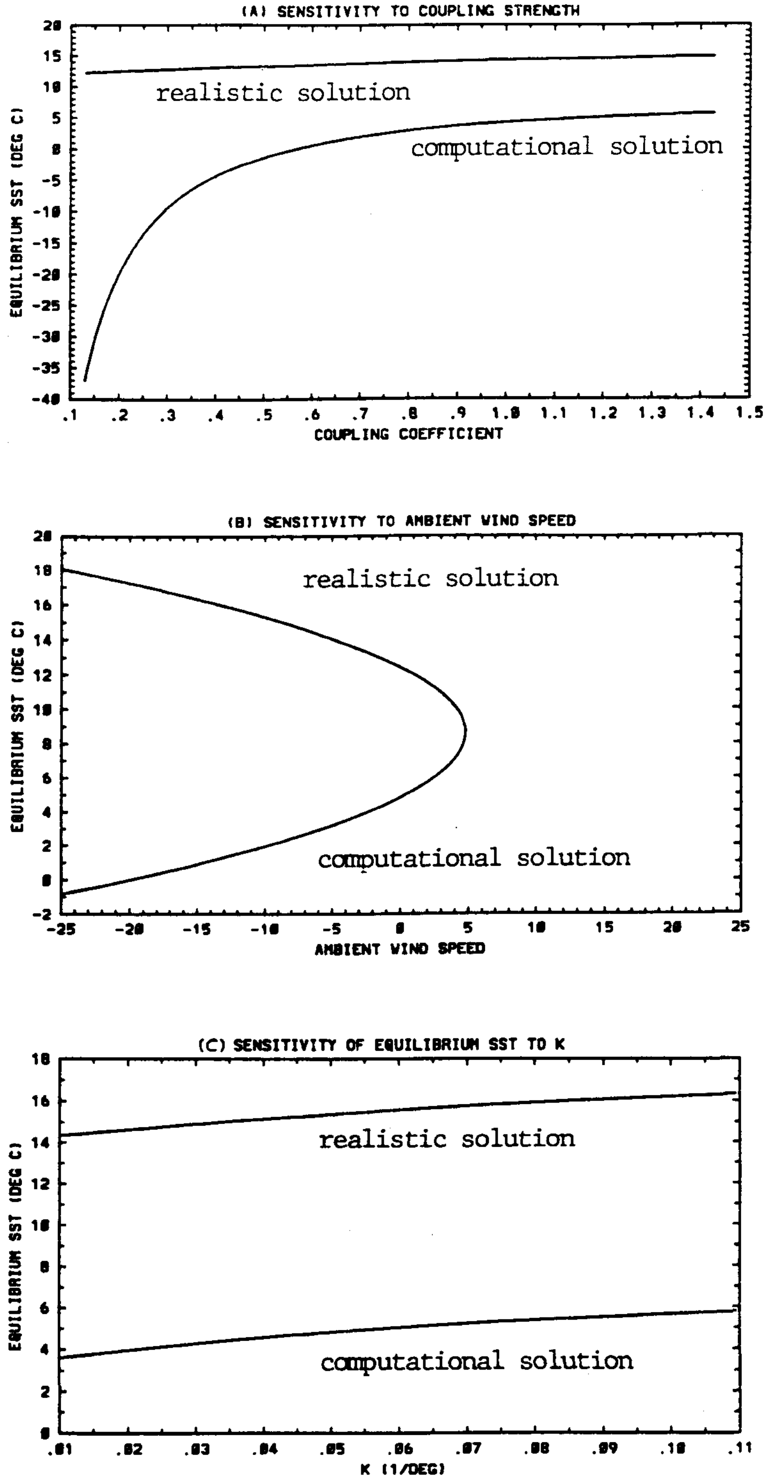

Fig. 2. Sensitivity of steady state solution to model parameters. a) Sensitivity to coupling strength. The coupling coefficient is represented by the value of A; b) Sensitivity to ambient wind speed (v_{am}); c) Sensitivity to Ekman depth variation induced by vertical temperature difference (represented by the value of k).